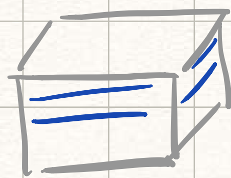
M^3 3-manif, closed hyp.

$\mathcal{F}$ co-oriented <u>taut</u> foliation



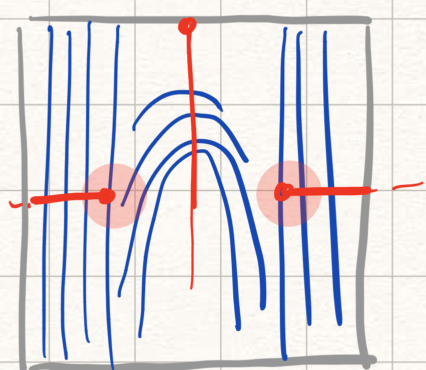
foliation = decomposing M into leaves

<u>Yesterday</u>: $\exists$ leaf-wise hyp. metric for $\mathcal{F}$.

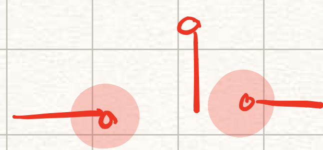
Take $(M, \mathcal{F}) \longleftarrow (\tilde{M}, \tilde{\mathcal{F}})$.

Let $L = L(\mathcal{F})$ leaf space for $\tilde{\mathcal{F}}$.

$\to L$ is <u>not</u> nec. Hausdorff 1-manif.



<u>Leaf space</u>



looks like this in leaf space.

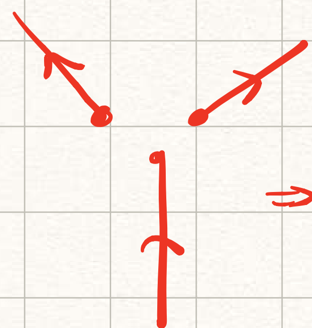
nbhd converges to two different types of leaves

A set of leaves $\{\lambda_j\}_{j \in J}$ forms a ==cataclysme== if $\exists$ a seq (μ_i) of leaves st

contained in a single embedded interval

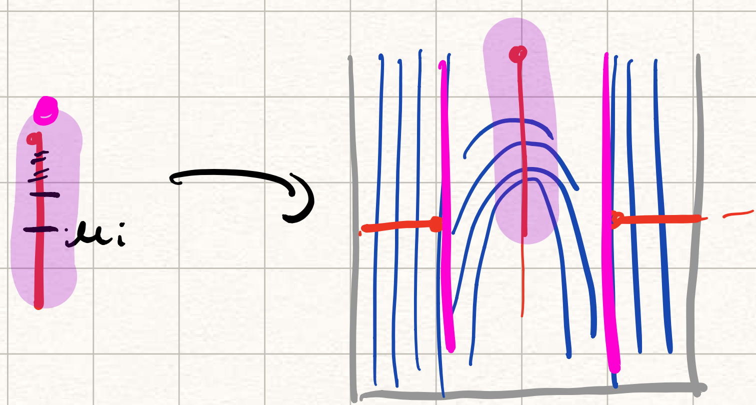
- μ_i forms a monotone seq wrt to induced partial order.

  → each interval has an induced orientation



⇒ gives partial order.

- in each compact subset K of $\tilde{M}$, $\mu_i \cap K \xrightarrow[i \to \infty]{} \lambda_j \cap K$ $\forall j$ in the Hausdorff top.



leaf space.

Claim: L simply connected.

Thm: (Rosenberg, Novikov)

If M is not finitely covered by $S^2 \times S^1$ & M admits $\mathcal{F}$ taut foliation,

then M is irred,

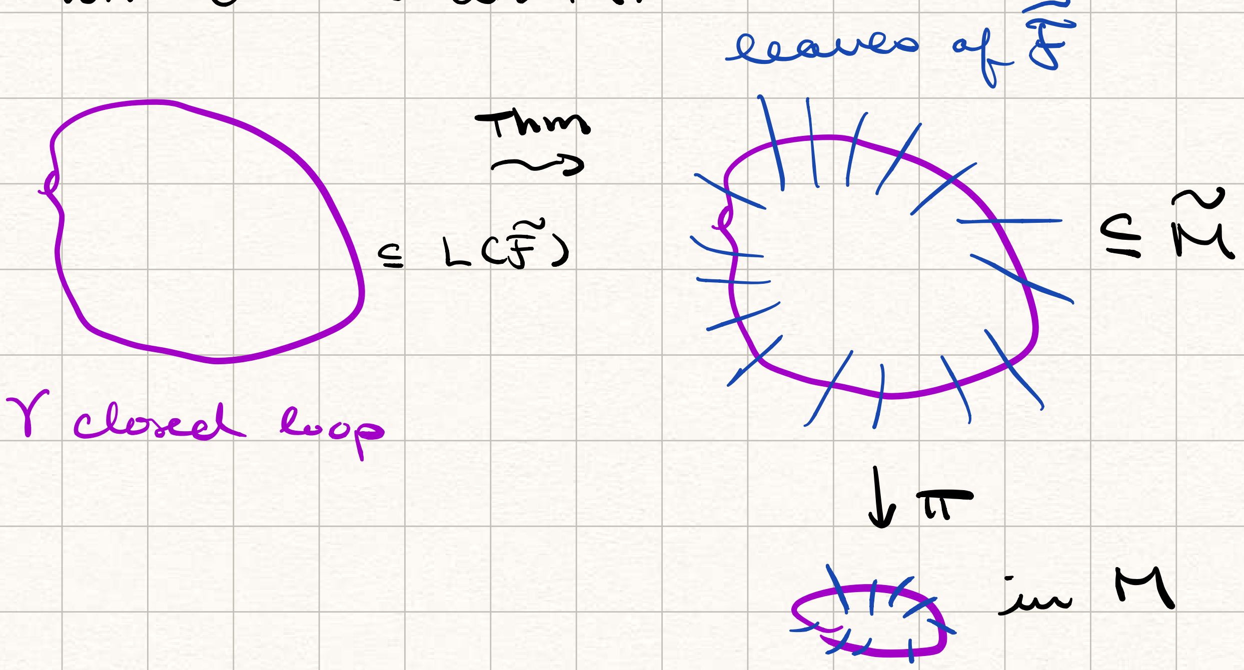
for each leaf α of $\mathcal{F}$ the inclusion map $\alpha \xrightarrow{i} M$

induces an inject

$$i_* : \pi_1(\alpha) \hookrightarrow \pi_1(M)$$

Pf Claim:

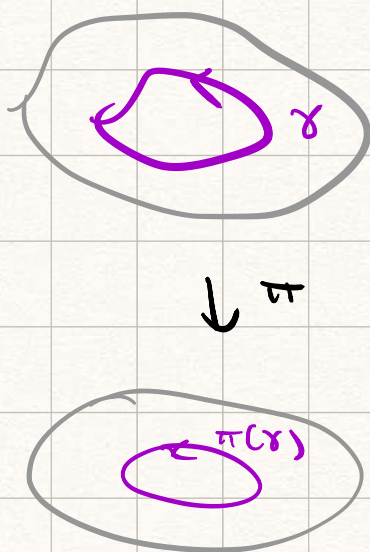
Every closed transversal to $\mathcal{F}$ is non-trivial in M.



Cor (of Thm)

$(M, \mathcal{F})$ as before. Every leaf of $\tilde{\mathcal{F}}$ is a properly embedded plane in $\tilde{M}$.

(pf): First, observe that leaves of $\tilde{\mathcal{F}}$ are simply connected.



$\tilde{\lambda}$ leaf of $\tilde{\mathcal{F}} \subset \tilde{M}$

λ leaf of $\mathcal{F}$.

γ trivial in $\tilde{M}$. ?
$\Rightarrow \pi(\gamma)$ triv in M
$\Rightarrow \pi(\gamma)$ triv in λ
$\Rightarrow \gamma$ triv. in $\tilde{\lambda}$.

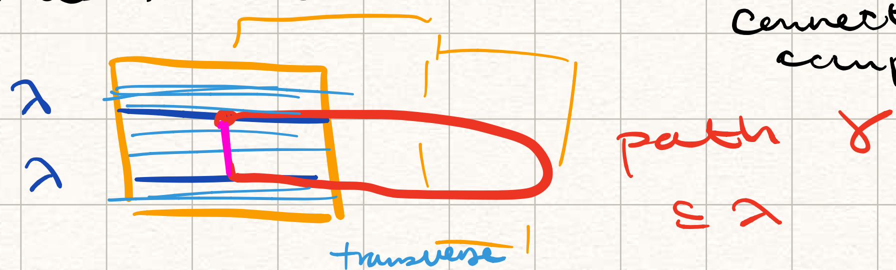
By Reeb stab, there are no spherical leaves.
$\Rightarrow$ leaves are planes.

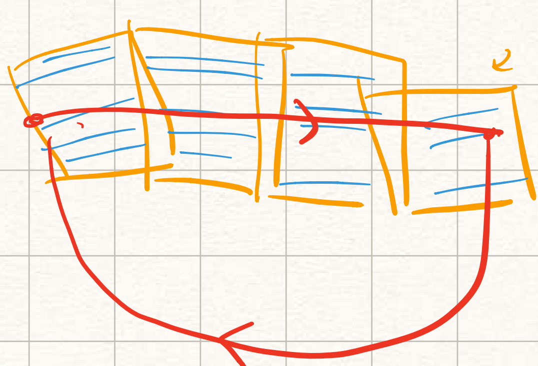
Let λ be leaf of $\tilde{\mathcal{F}}$.

Obs: If λ is covered by prod charts, so that in each chart, intersection w/ λ is connected.

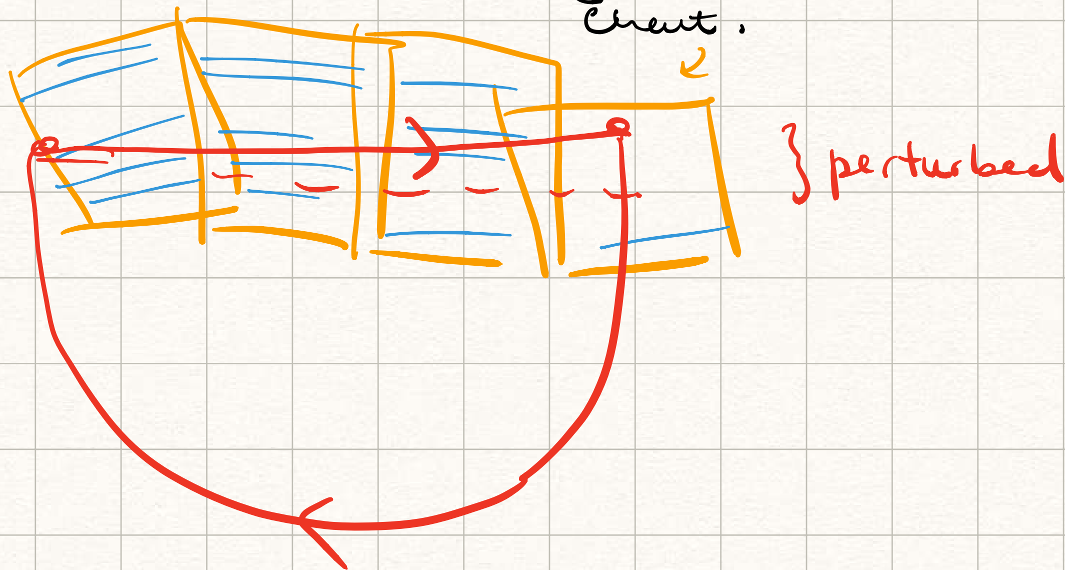
Then, λ is properly embedded.

Pf: Suppose that λ is not properly embedded. Then $\exists$ prod. chart where λ meets at more than one connected component.

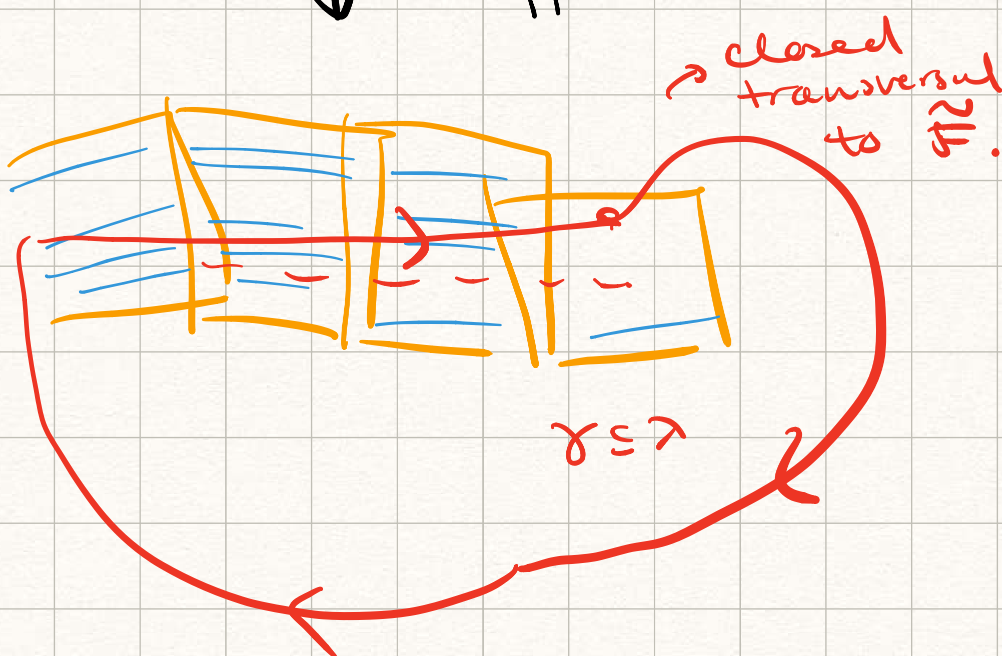




↓ perturbe, now whole thing is transverse wh chart.



↓ but this can't happen.



picture must be this

Yeah I don't understand, but also don't think I can.

Cor: $(M, \mathcal{F})$ taut leafwise hyp covering from Candel. *

$\rightsquigarrow (\tilde{M}, \tilde{\mathcal{F}})$ each leaf of $\tilde{\mathcal{F}}$ is isom to $\mathbb{H}^2$ wrt induced path-metric.

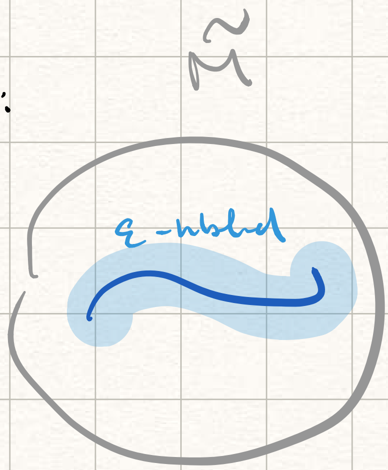
* He said they pf'd no spherical, or torus leaves $\Rightarrow$ hyp. by Candel.

For each leaf λ of $\tilde{\mathcal{F}}$, we get a circle at infinity $S^1_\infty(\lambda)$.

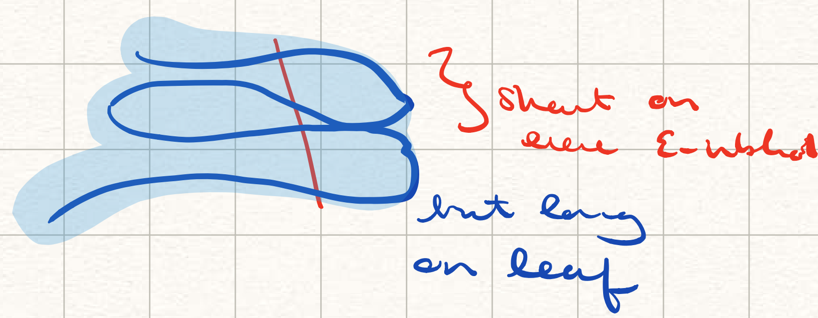
Let $E_\infty = \bigcup_{\lambda \in L} S^1_\infty(\lambda)$

Transverse Geom of $\tilde{\mathcal{F}}$

Pf: $\exists \varepsilon > 0$ st every leaf λ of $\tilde{\mathcal{F}}$ is q-isom embedded in ε-nbhd ex:



non-ex



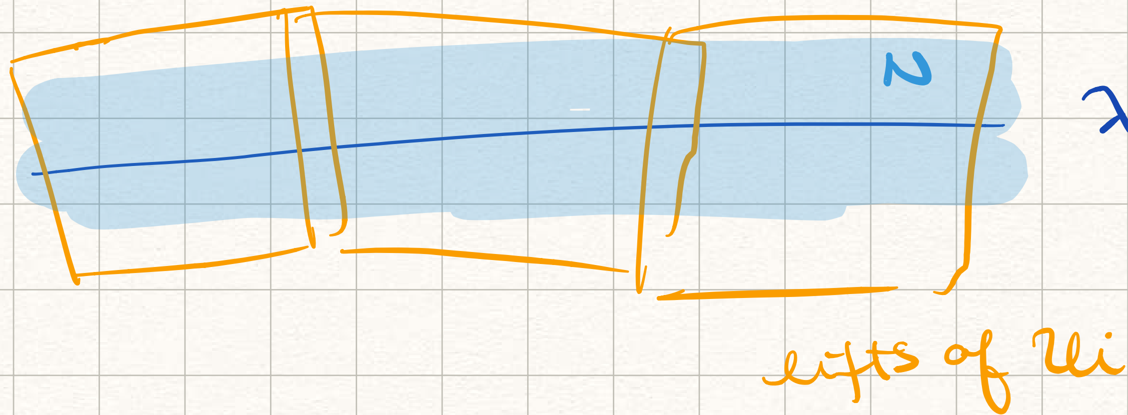
Pf: $\forall p \in M$, $\exists\, \mathcal{U}_p$ nbhd of p st

- evenly covered wrt $\tilde{M} \xrightarrow{\pi} M$
- each slice of $\pi^{-1}(\mathcal{U}_p)$ is a prod chart where each leaf of $\tilde{\mathcal{F}}$ intersect at most once.

M cmpt $\Rightarrow$ we can cover it w/ finitely many such $\mathcal{U}_1, \dots, \mathcal{U}_n$.

By Lebesgue number lemma, $\exists\, \varepsilon > 0$ st every ball of radius 2ε is constrained in some $\mathcal{U}_i$.

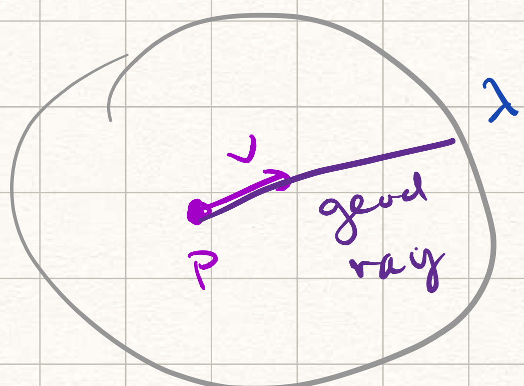
Let λ be leaf of $\tilde{\mathcal{F}}$. $N = N_\varepsilon(\lambda)$.



- Such ε is called separation ct of $\mathcal{F}$.

- Endpt map $p \in \lambda \subseteq \widetilde{M}$ and $v \in T_p \lambda$

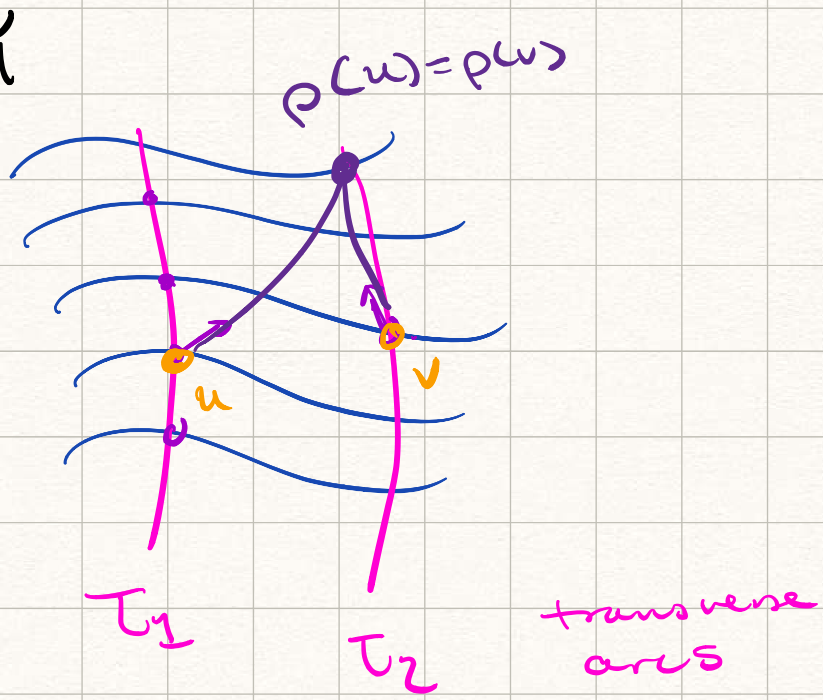
  leaf of $\widetilde{\mathcal{F}}$



$$e : UT\widetilde{\mathcal{F}} \longrightarrow E_\infty$$

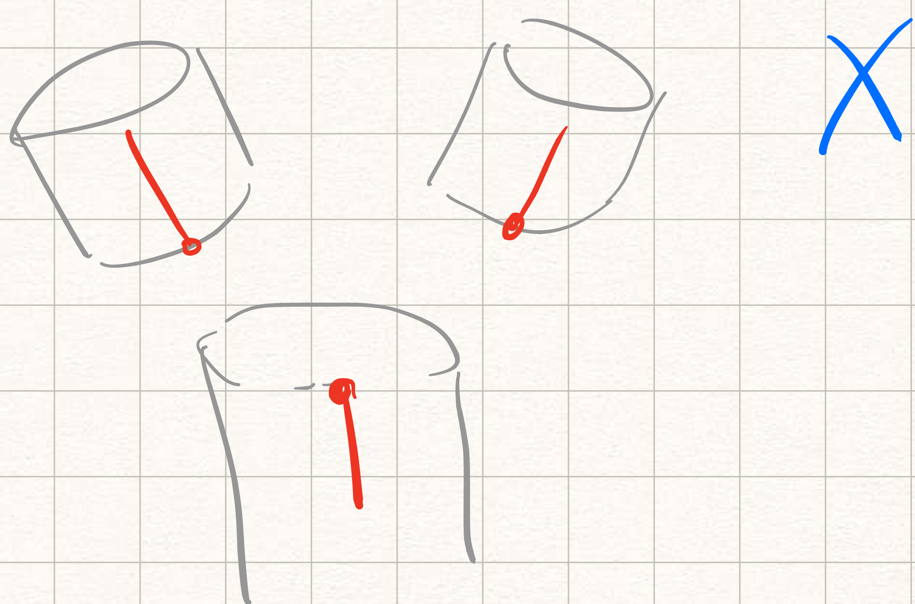
One way to give a top on E_∞ is just giving the finest top so that e is cont.

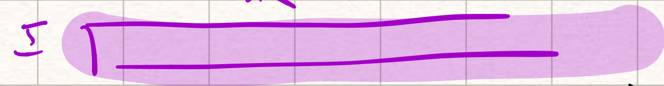
In $\tilde{M}$



$p : \tau_1 \to E_\infty$.

Consider the cylinder
$C(\tau_1) = p(\tau_1) \subset E_\infty$

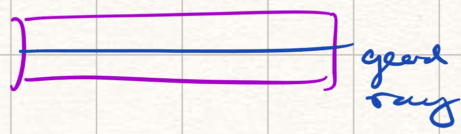




Consider $m : I \times \mathbb{R}_{\geq 0} \longrightarrow \tilde{M}$.

st: • for each $t \in I$,

$m|_{\{t\} \times \mathbb{R}_{\geq 0}}$ is a geodesic ray in some leaf.



• for each $s \in \mathbb{R}_{\geq 0}$,

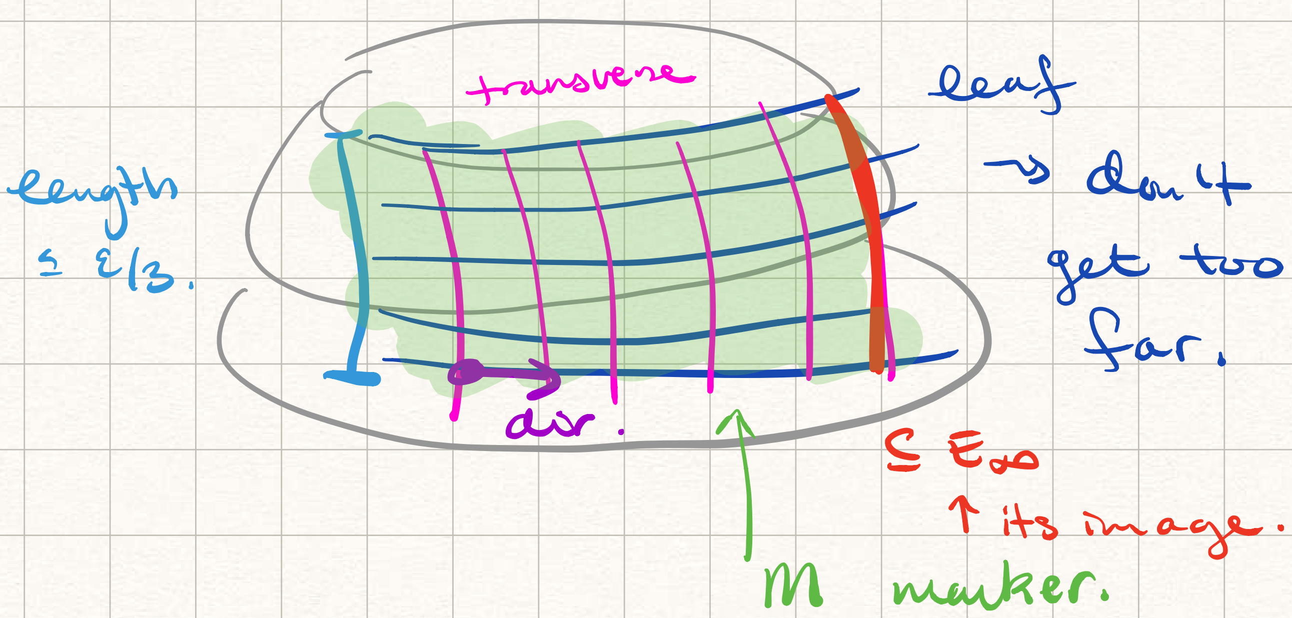
$m|_{I \times \{s\}}$ is a transverse ray.

of length (in $\tilde{M}$) $\leq \varepsilon/3$,

where ε is some chosen separation ct.

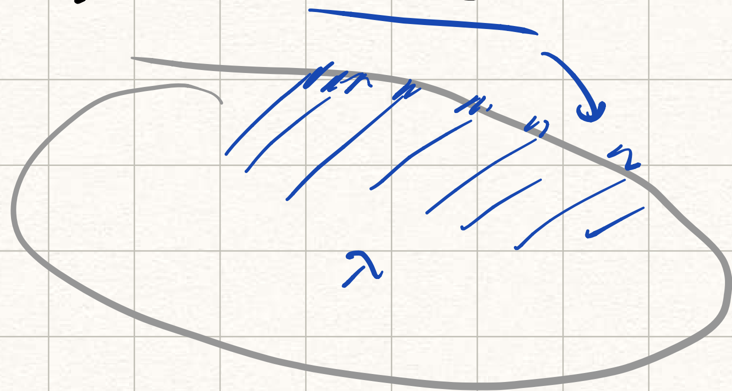
Such m is called a marker.

(We'll also call the image of m on E_∞ a marker).



Thm (The leaf pocket thm)

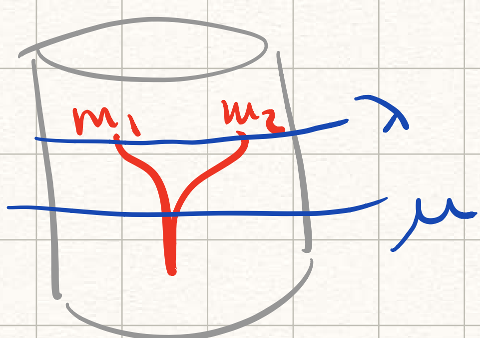
The set of marker in each $S'_\infty(\lambda)$ is dense.

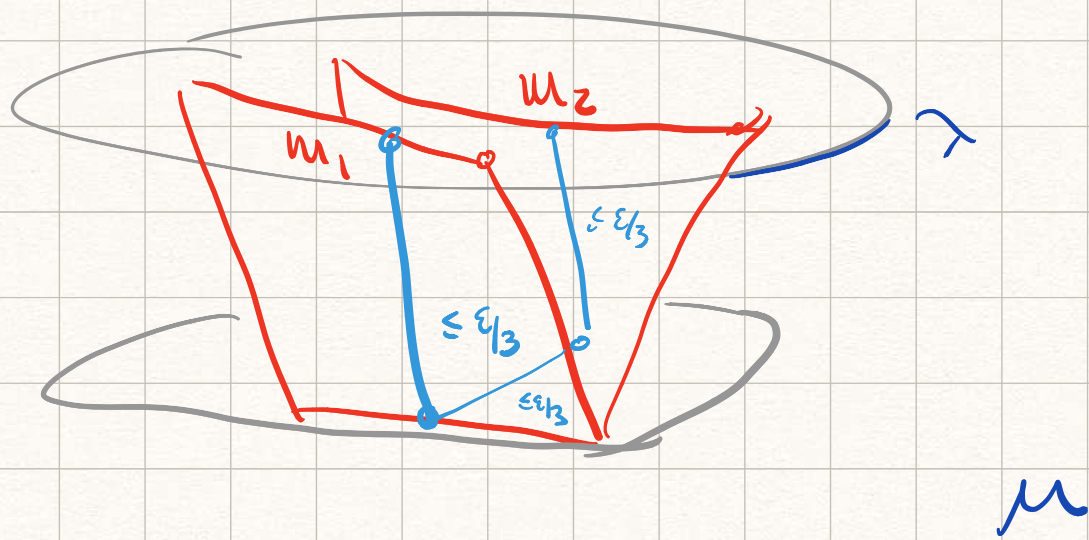


Claim: $I \subset L$ interval.

$$C = \bigcup_{\lambda \in I} S'_\infty(\lambda) \subseteq E_\infty .$$

Let m_1, m_2 be two markers in C. Either $m_1 \cap m_2 \neq \emptyset$ or $m_1 \cup m_2$ forms another marker.





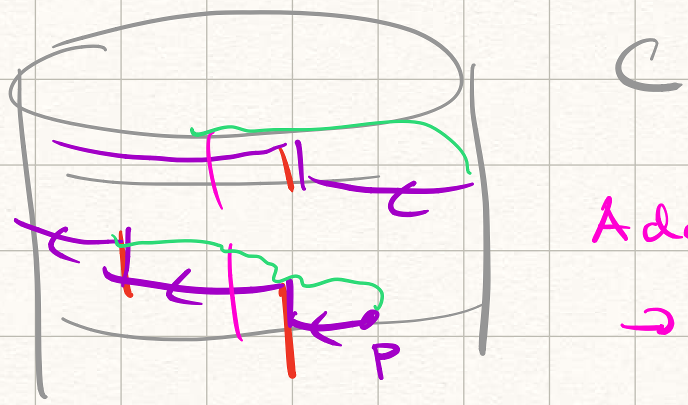
We may assume that
$d(x,y) \le \varepsilon/3$ for $x \in M_1 \cap \mu$
$y \in M_2 \cap \lambda$

Since M_1, M_2 go to two dif pts on $S_\infty(\lambda)$, they get arb.
taut

far apart in λ.

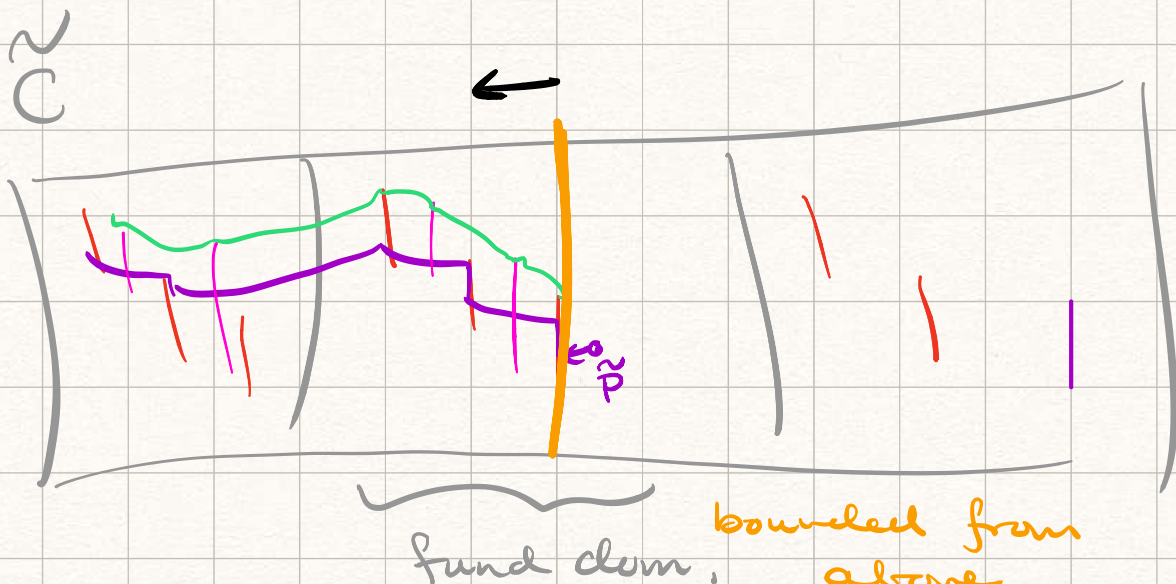
<u>**But**</u>: in ε-nbhd of λ, we know dist is short. Contra.

Consider:



Add markers.
→ get new paths

$\mathcal{M}$ set of markers.
leftmost path from p wrt $\mathcal{M}$. $T_{\mathcal{M}}$
New path.



⇝ Should get some partial ord.

For each $v \in I$, exists b/c all $T_{\mathcal{M}}$'s bounded from above

$$\sigma(v) := \sup_{\mathcal{M}} \left(\min T_{\mathcal{M}} \cap S_{\infty}^{1}(v) \right)$$

ordering

We get σ to be a continuous section of $\underline{C}$, S-bundle over S^1.