V - Elementary amenable grps

$EA_0 := \{G \mid G \text{ is ab or finite}\}$

For ordinals $\alpha > 0$,

$$EA_\alpha := \left\{ G \mid G \begin{array}{l} \text{is an ascending} \\ \text{union or an extension} \\ \text{of grps from } \bigcup_{\beta<\alpha} EA_\beta \end{array} \right\}$$

$N \hookrightarrow G \twoheadrightarrow Q$

exc: Show that each EA_α is closed wrt to taking subgrps and quotients

Thm: F is NOT elementary amenable.

Pf: Otherwise, choose α minimal w/ $F \in EA_\alpha$. F is neither finite nor ab. Thus, $\alpha > 0$.

F is fg
$\Rightarrow$ F does not enter EA_α as an asc. union.

thus; $N \hookrightarrow F \twoheadrightarrow Q$, where

$N, Q \in \bigcup_{\beta<\alpha} EA_\alpha$.

<u>case 1</u>: $F \hookrightarrow [F,F] \hookrightarrow Q \in EA_\alpha$

$\Rightarrow F \in EA_\beta$ for $\beta < \alpha$.

re-listen, contra.

<u>Case 2</u>: $F \hookrightarrow [F,F] \hookrightarrow N \in EA_\beta$

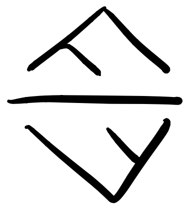
same.

<u>Open Problem</u>: Is F amenable?

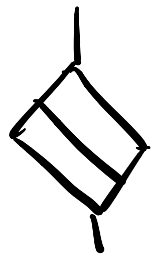
<u>Rmk</u>: F does not contain non-abelian free grps! (I guess that's why ppl care).

VI - Strand diagram & Stein Space

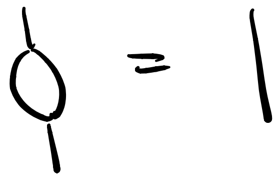
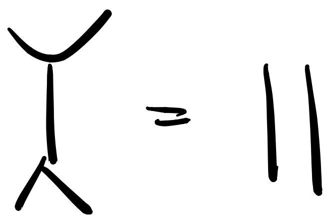
Idea: Rewrite



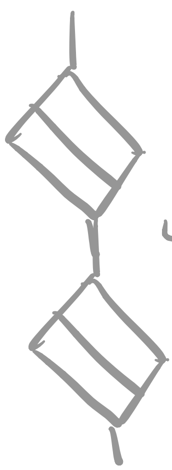
as



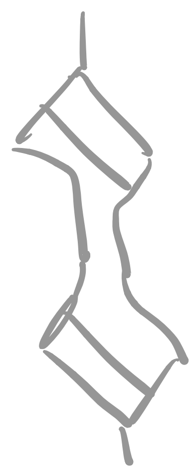
Rules:

-  eye removal
-  x-removal

w/c



wants to be



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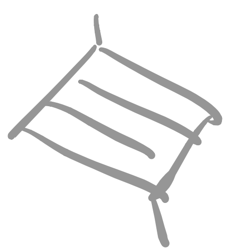
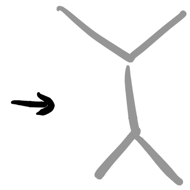
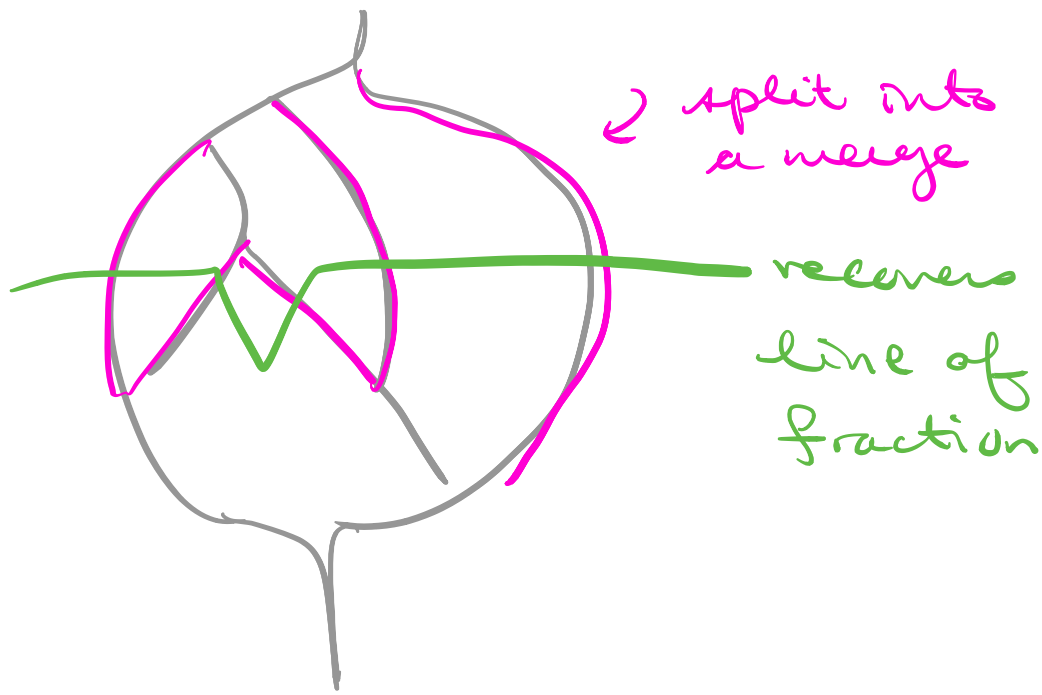


A diagram is reduced if there's no eye or X removal possible.

Obs: X removal possible
⇔ there's a strand along which a split follows a merge.

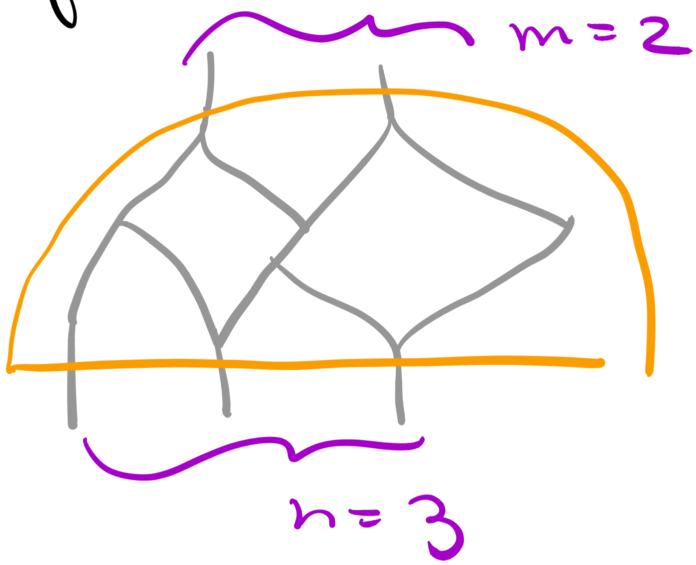


Reduced ⇒ along each strand, you first encounter all the splits, then all the merges.

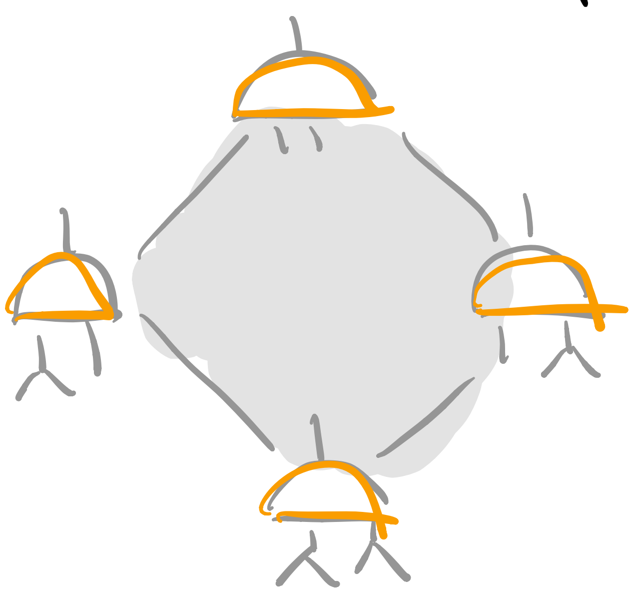


Def: An (m,n)-diagram is a strand diagram w/ m heads and n feet.

is a bell to contain mess.



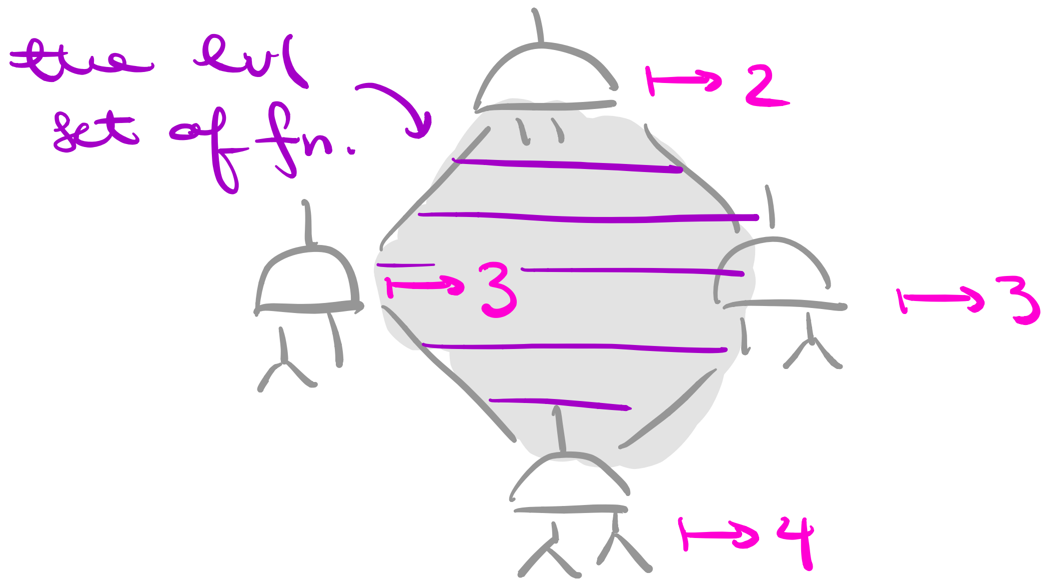
Obs: The $(1,*)$-diagrams can be arranged to become the vertices of a cube complex.



There is a Morse function

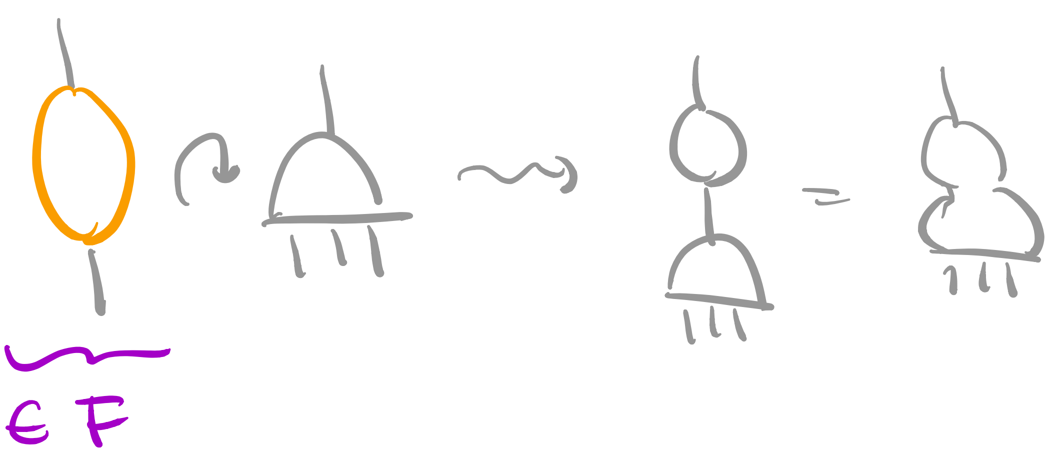
$h: X \to \mathbb{R}$

$v \mapsto \#$ feet in v



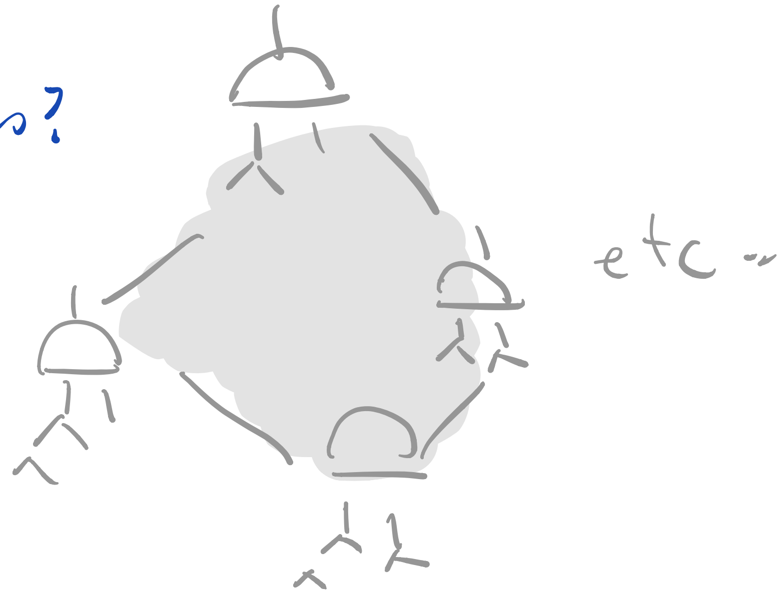
Let X_m: subcomplex spanned by vertices w/ $\leq m$ feet.

Obs: F acts on X from above



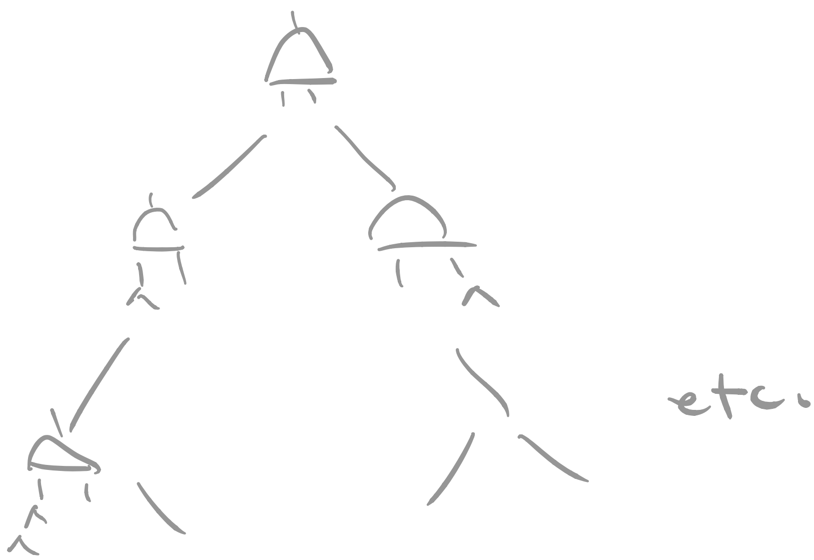
Q: more cubes?

A: Yes.



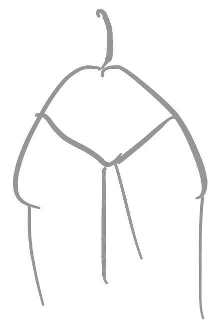
Lem: X is contractible.

Pf Sketch: Every vertex $v \in X$ is ~~the~~ root at a "cone" X_v of everything obtained from v by splitting feet over & over.



$\Rightarrow X_v$ is contractible towards v.

1. X is covered by the X_v.
2. $X_v \cap X_w = X_{v \wedge w}$.



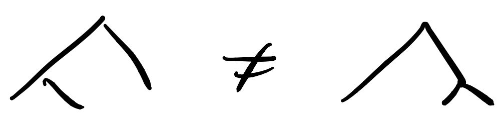
didn't quite get it but split & merges disappear eventually.

Nerve-Cover thm

X is homotop equiv to the simplices over all vertices v.

Let $\mathcal{N} = \{\sigma : \text{sets of verts } \bigcap_{v \in \sigma} X_v \neq \emptyset\}$

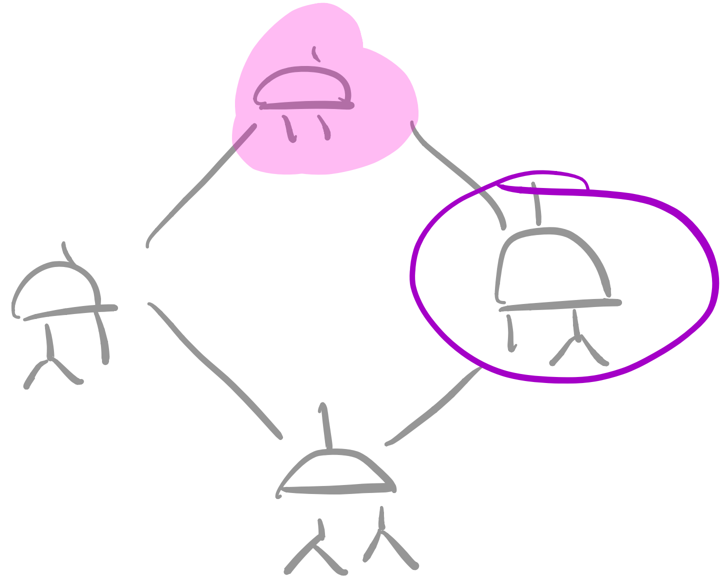
N.B :



but

this is how we get intersects.

Descending links



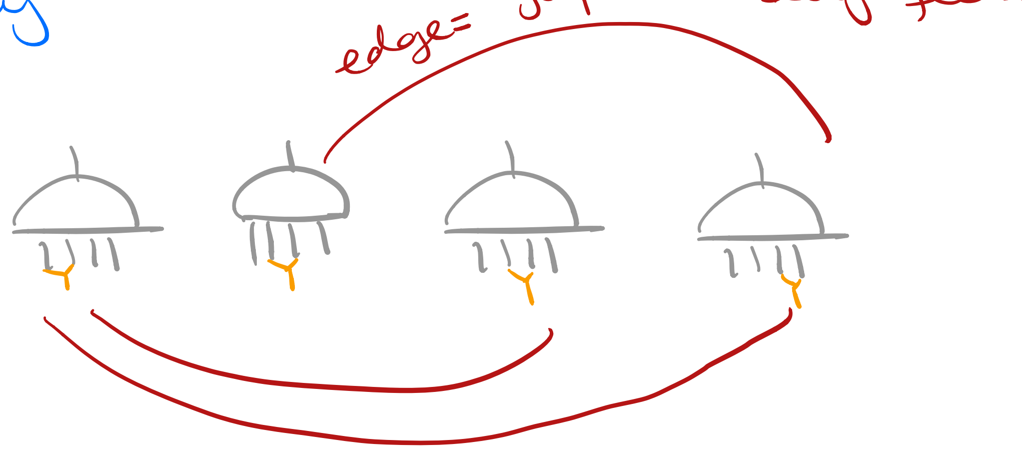
$= v$

? not sure about order

$w \in \overset{\downarrow}{lk}(v) \iff v$ can be obtained from w by splitting one foot

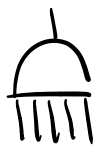
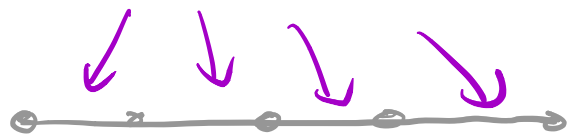
? why

ex :



$lk^{\downarrow}(v) =$ matching complex of a linear graph w/ # feet nodes

(positions for possible Y's)

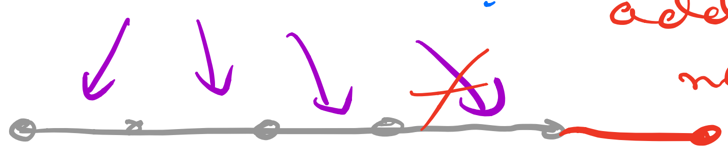
$lk^{\downarrow}($  $) =$ 

Simplices = collection of disj. edges.

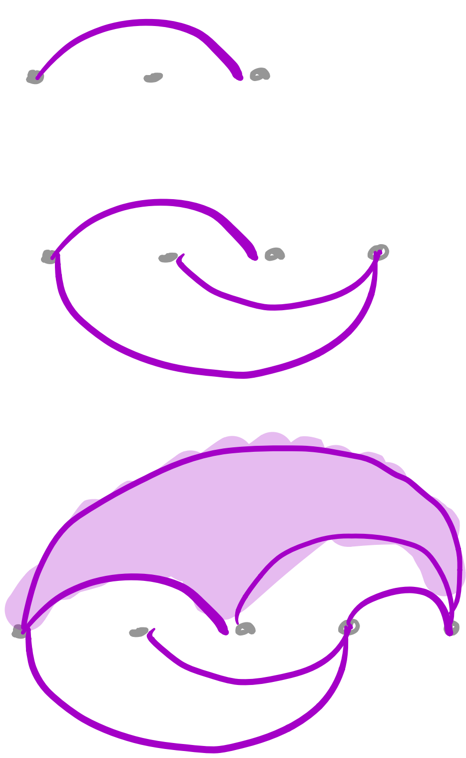
Sequence:

$S^0, S^0 * S^1, S^1 * S^2, S^2 * \dots$

? not sure about this



adding one new is compatible w/ everything but this guy.



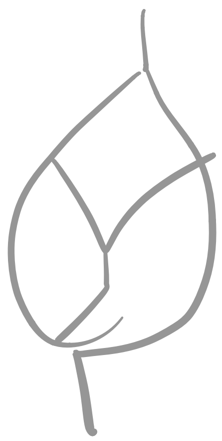
homotopy type of these

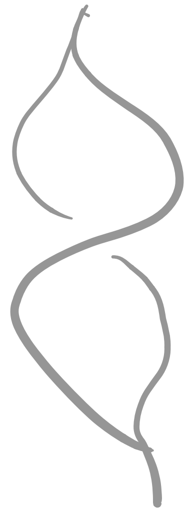
Brown, Bestina–Brady

F is of type F_∞:

- control over cell stabilizers
- cocompact filtration
- relative connectivity / descending lks.
- contractible space.

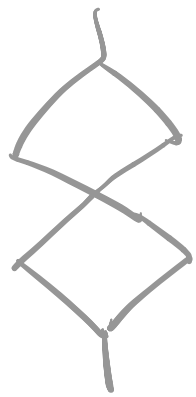
Relatives :





re-listen

split
braid
merge



split
permute
merge

$\to V_{br} \rightsquigarrow$ braided V

$\rightsquigarrow V \subseteq \mathrm{Homeo}(\mathcal{C})$.

Thm: V is simple & of type F_∞.