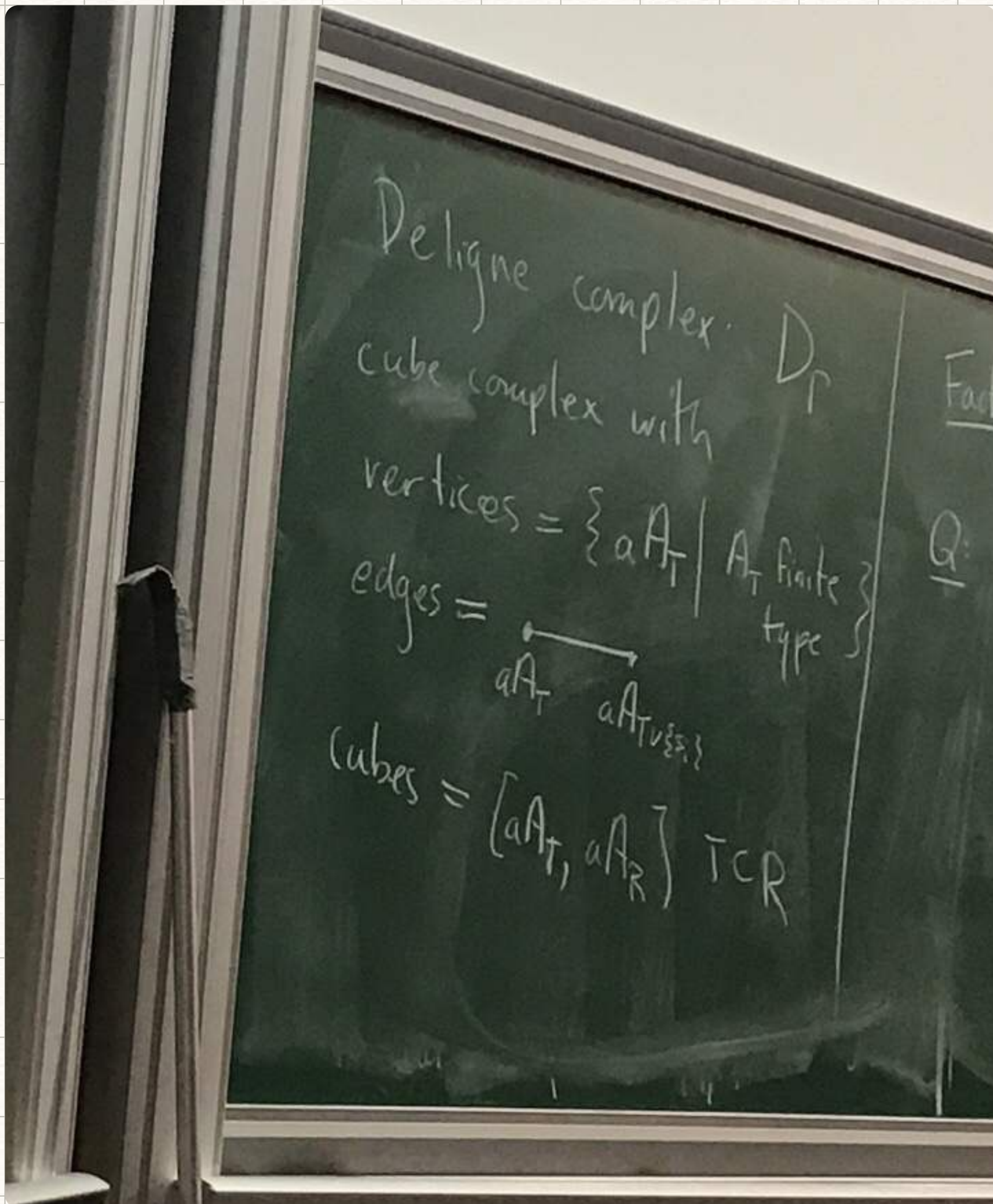


Fact: $D_\Gamma \simeq \tilde{\mathcal{H}}/\Gamma$

Q: Is D_Γ npc?

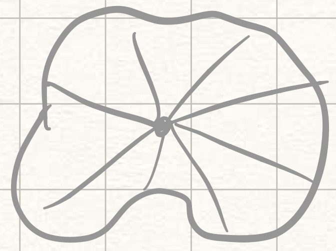
Recall: X cc. geod. m.s.

X simply connected + npc.

$\Rightarrow X$ CAT(0)

$\Rightarrow$ geodesics are unique

$\Rightarrow$ X contractible.



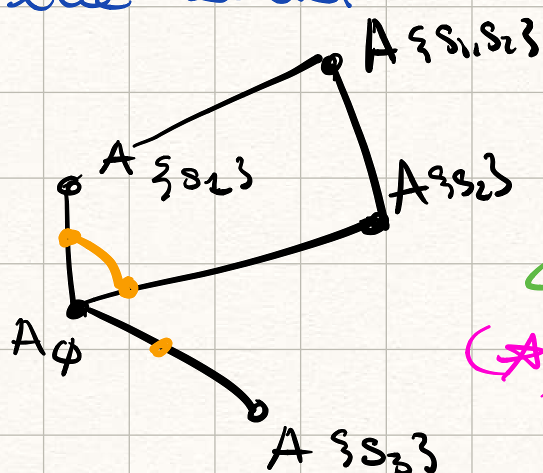
Need to show that D_Γ is npc.

D_Γ npc $\Leftrightarrow$ link of vertices are flag-complex.

flag complexes are simplicial complexes w/o empty simplices.

Consider the vertex $x = A_\emptyset$ in D_Γ

What is the link?



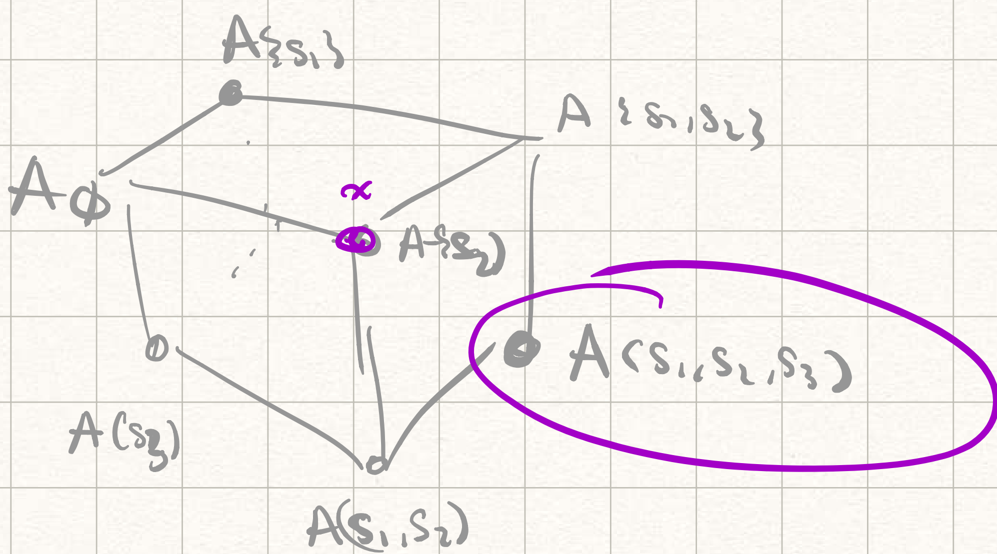
allowed if $A_{\{s_1,s_2\}}$ finite type subgr'p $\Leftrightarrow$ $m_{12} < \infty$

$(\ast)$

$\star \iff \underset{s_i}{\bullet}\!\overset{m_{ij}}{\text{———}}\!\underset{s_j}{\bullet} \quad m_{ij} < \infty$

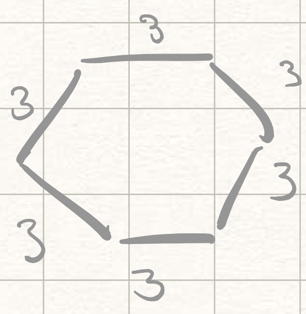
Thus ~~the~~ 1-skeleton of lk(x) is exactly Γ.

We need to check whether every clique in Γ is filled w/ a simplex in lk(x)

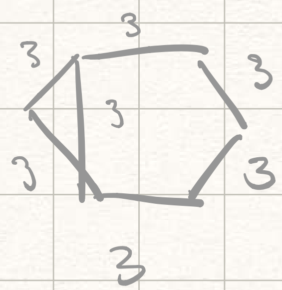


$\Rightarrow$ lk(x) is flag $\iff$ every clique T in Γ
($\star$) is st A_T is of finite type

We say that A_Γ is FC-type (flag complex type) if $\star$ holds.

eg: $\Gamma =$ 

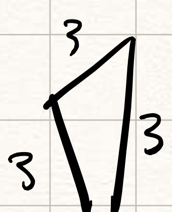
①

$\Gamma =$ 

②

$\Gamma =$ any RAAG. ③

① is FC-type b/c all cliques $\longrightarrow$

② 

= affine coxeter, NOT finite type.

③ Yes (re-listen).

Thm: D_Γ is CAT(0) $\Longleftrightarrow$ A_Γ is FC-type.

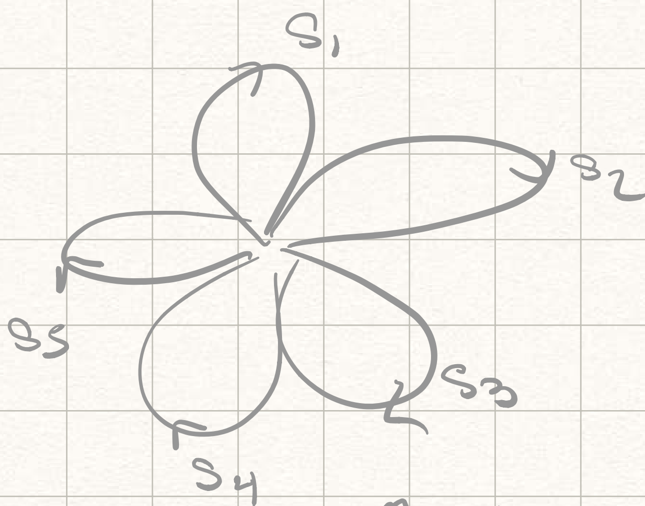
$\Downarrow$

what's this? $K(\pi,1)$ conjecture holds for A_Γ.

Thm (Speaker)
If A_Γ is FC-type, then <u>all</u> of the conjectures (old conj) hold.

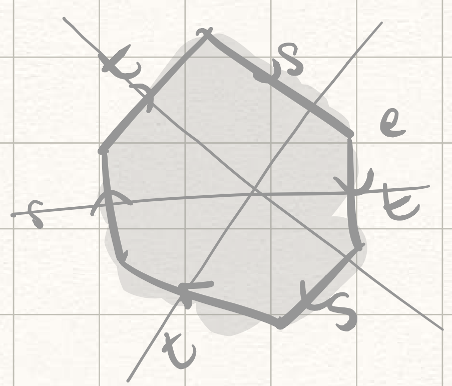
Rmk: The finite $K(\pi,1)$-space is a generalization of the Salvetti complex for RAAGs.

$S_\Gamma =$



glue

$T \subseteq S$, W_T



re-listen
Coxeter Sail.

Obvious that $\pi_1(S_\Gamma) = A_\Gamma$,
but need to show universal cover
is contractible for $K(\pi, 1)$.

To prove conj 4, show
$\tilde{S}_\Gamma \simeq \tilde{H}_\Gamma$.

? why

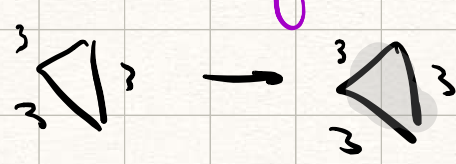
So conj (5) $\Rightarrow$ conj (4) $\Rightarrow$ conj (2)

So now suppose A_Γ not FC-type.

We can modify D_Γ to make it
CAT(0) $\forall \Gamma$.

The problem was that cliques
weren't filled due to
certain A_T not being finite
type.

(just don't require finite type!

ie 

Define clique cc C_Γ.

vertices = $\{aA_T \mid a \in A_\Gamma, T \subseteq S$ spans a clique$\}$

edges + cubes as before.

Thm C_Γ is CAT(0) for all Γ.

Problem:

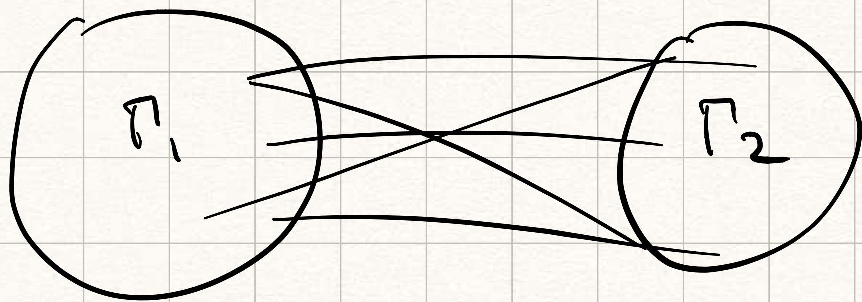
- No longer know that $C_\Gamma \simeq \mathcal{D}_\Gamma$.
- If Γ itself is a clique then $\operatorname{diam}(C_\Gamma) < \infty$.
  In fact, every max'l cube (maximal cube) contains $A_S = A_\Gamma$.

Thm: If Γ is not the star
1) of a single vertices, then A_Γ has trivial center.

2) If Γ is not a <u>join</u>, ($\Gamma \neq \Gamma_1 * \Gamma_2$)
ie $\Gamma = \Gamma_1 * \Gamma_2$, then
A_Γ is acyl. hyp.

3) All the other conj reduce
to the case where
Γ is a single clique.

<u>Rmk</u>: $\Gamma \neq \Gamma_1 * \Gamma_2$ refers to
Γ <u>unlabeled</u> graph.



Would like $A_\Gamma \neq A_{\Gamma_1} \times A_{\Gamma_2}$
~~to~~ for the edges to be

$\overset{2}{\text{———}}$ w/ 2's.

... can't deal w/ clique case.