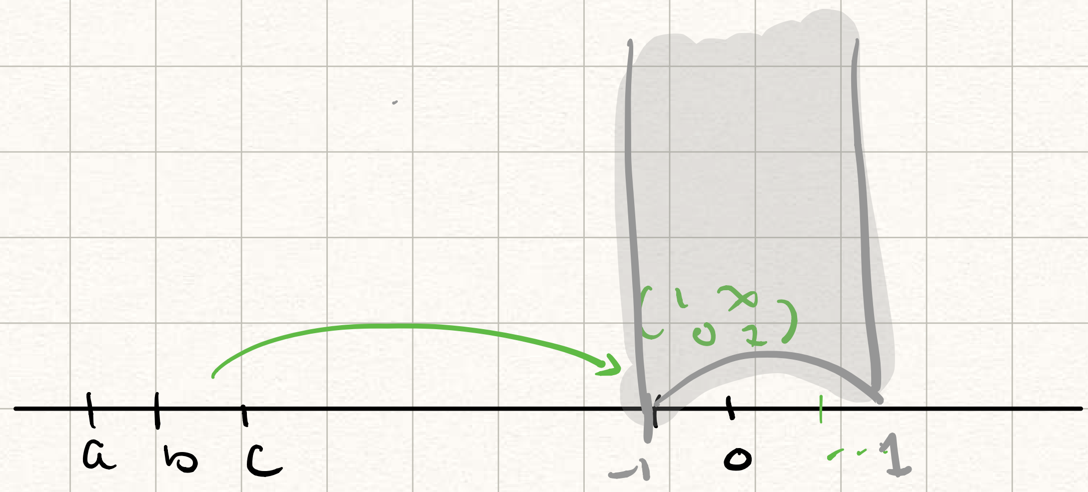
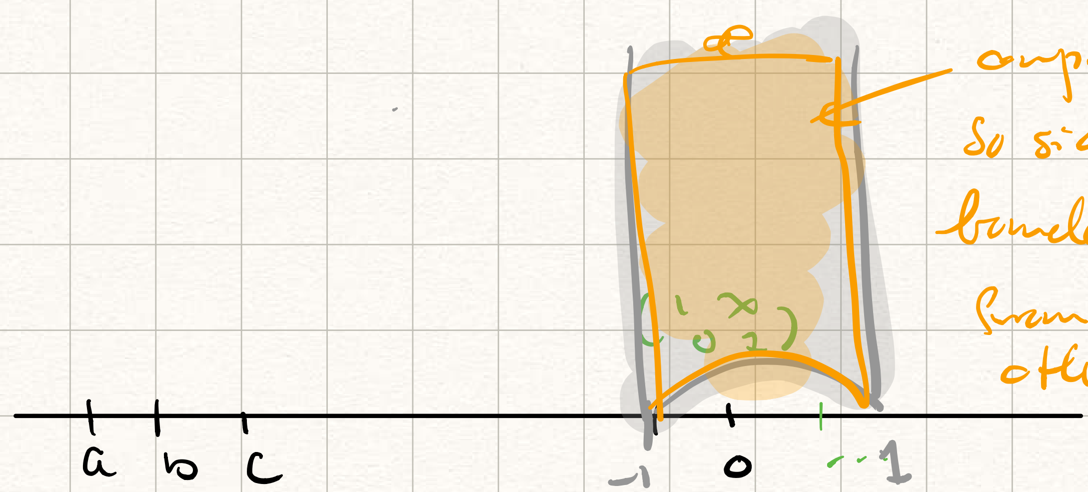
exc 1: $SL_2\mathbb{R} \curvearrowright$ triples of $\mathbb{R}\cup\{\infty\}$

→ Take $\begin{pmatrix}1&x\\0&1\end{pmatrix}, \begin{pmatrix}0&1\\-1&0\end{pmatrix}, \begin{pmatrix}\lambda&0\\0&\lambda^{-1}\end{pmatrix}$



... Ok (you've seen this before)

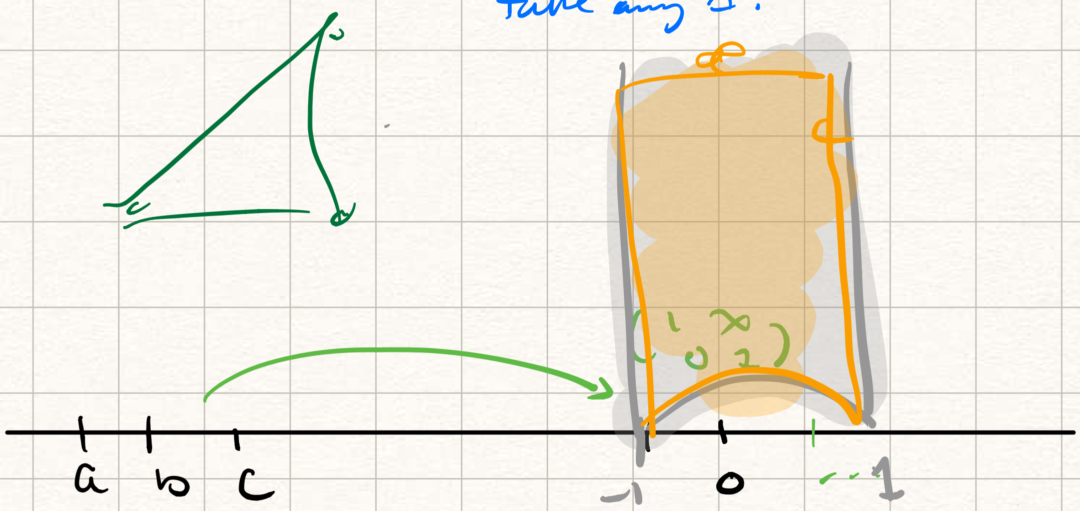
exc 2



compact, so side bounded from each other

exc 3

re-listen,
take any Δ.



exc 4:

$$\mathbb{R}^n \xrightarrow{qi} \mathbb{Z}^n$$

$$(x_1, \dots, x_n) \longmapsto (\lfloor x_1 \rfloor \dots \lfloor x_n \rfloor)$$

exc 5:

$$\mathrm{Cay}_{S_1} G \overset{qi}{\simeq} \mathrm{Cay}_{S_2} G$$

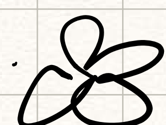
by $\max\limits_{s \in S_2} d_{S_1}\{1, s\}$.

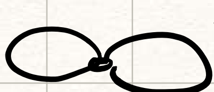
exc 6:

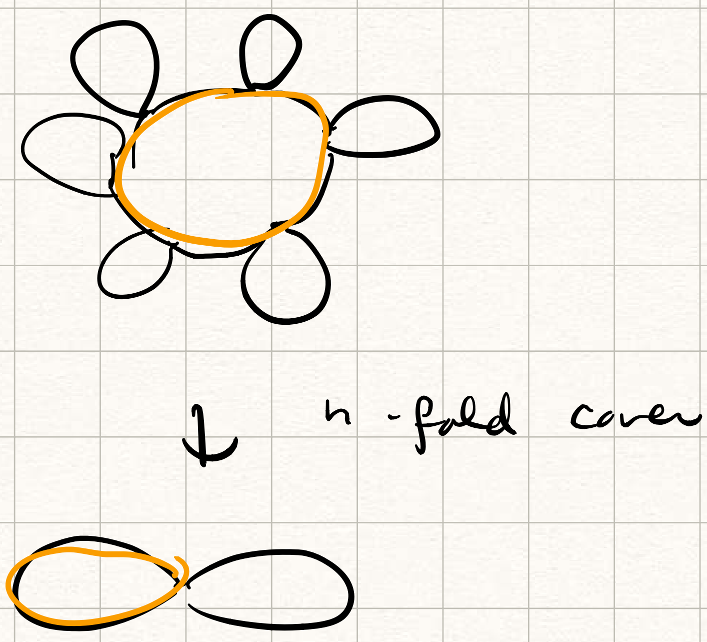
$H < G$ fi

$H \curvearrowright \mathrm{Cay}\ G$
freely & co-compact $\Rightarrow$ qi.

exc 7:

$\mathbb{F}_k = \pi_1($  $k)$

$F_2 = \pi_1($  $)$



$\Rightarrow$ finite rk cover,
i.e. $\mathbb{F}_k \underset{fi}{<} \mathbb{F}_2$.

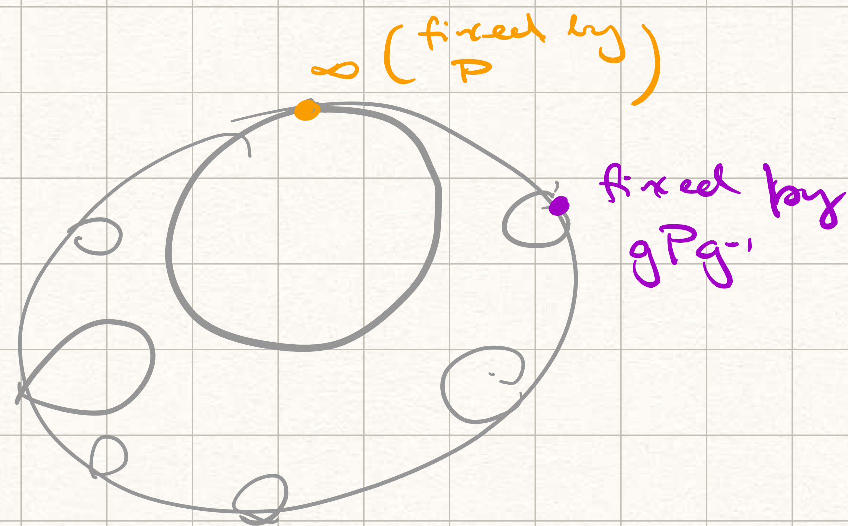
<u>exc 8</u>: $\mathbb{Z}^2$ not qi $\mathbb{Z}^3$
use growth fns; they are
q-isom invariant.

Last time: Hyp. grps & rel. hyp. grps

rel. hyp = · hyp. "fine" graph

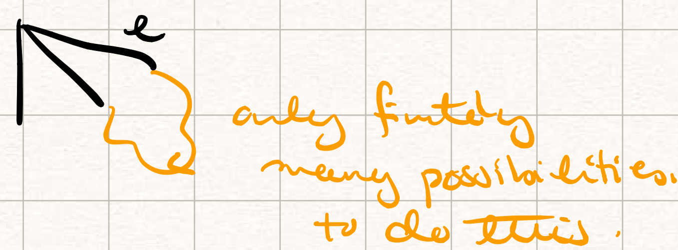
OR · proper hyp. spaces w/ invariant sys of horoballs.

horo:

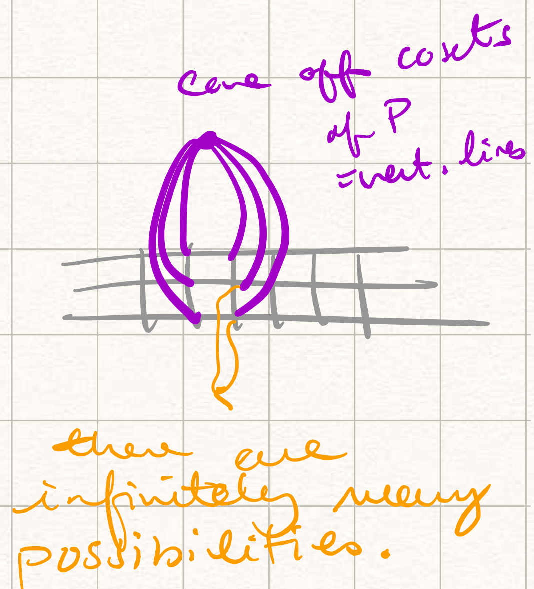


G, $P < G$

"fine"



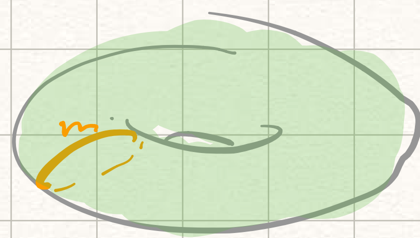
Non-ex: $\mathbb{Z} \oplus \underbrace{\mathbb{Z}}_{P}$



V - Dehn Filling thm

Let M^3 3-manifold w/ boundary torus,
$\partial M^3 = T^2$. (ex: Sphere − trefoil knot)

Goal: fill boundary.



Solid Torus (ST)

Choose a slope in T, glue ST st m goes on the slope.

Thurston: If M = complete finite vol hyp. manifold w/ one cusp of dim3, then for all but finitely many slopes, $M \underset{s}{\cup} ST^*$, $s \in \{\text{slopes}\}$ is a hyp. compact 3-manif.

$*$ = ST glued on M st m goes on slope s.

Thm (Osin + G-M)

If G is hyp. rel. P, there exists a finite set $\mathcal{F} \subset P\setminus\{1\}$ st $\forall N \triangleleft P$ avoiding $\mathcal{F}$, the grp $\langle\langle N\rangle\rangle^G$ contains a faithful image of $N\backslash P$ and is hyp. rel to $N\backslash P$.

but not off-hand comment :)

I dk what slope he mean.

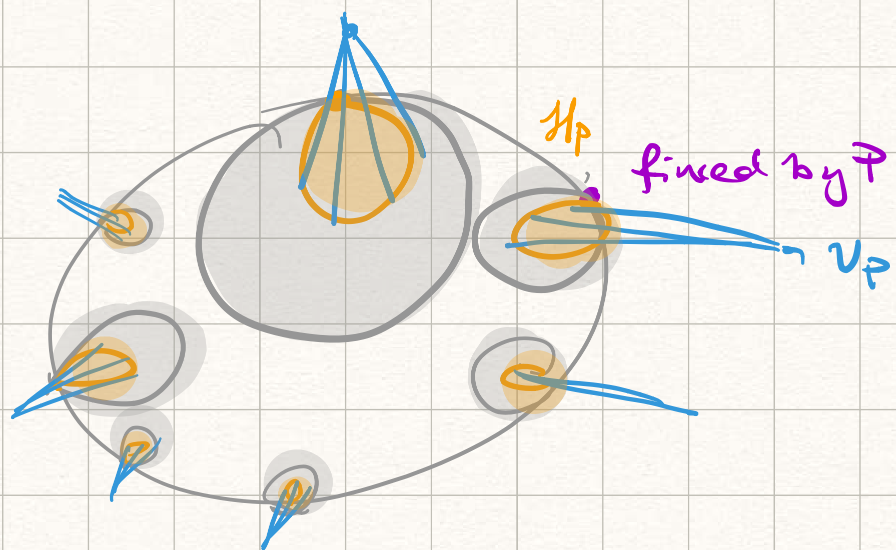
ex: take $\mathbb{Z}^2$, take $\mathcal{F} \subset \mathbb{Z}\setminus\{1\}$. Take cyclic subgp of $\mathbb{Z}^2$ taking slope not in $\mathcal{F}$, then $\mathbb{Z}^2_{\langle\langle N\rangle\rangle}$ hyp rel to $\mathbb{Z}^2_{\langle\langle N\rangle\rangle}$.

(**Fact** (Osin): If G hyp. rel. P, and P is hyp, then G is hyp.)

In his comment, we assume this implicitly.

Sketch pf theorem:

Take X = the model w/ invar. horoballs. (G, P)



orange = "deep" sys of horoballs.

blue = cone off horosphere only.

= boundary horoball

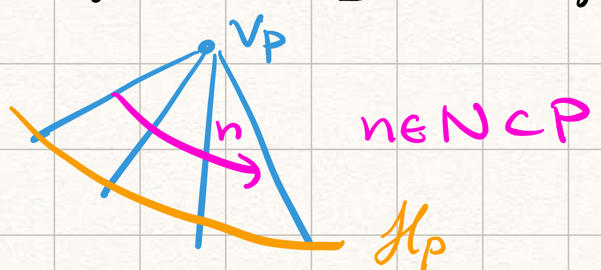
- Now cone off the horoballs
- P act on horoballs

But w/ almost angle zero b/c P fixes pt on ∂G, and take v "deep" horoball.

P acts by rotation around v_P. ← here action not nec. "almost trivial" like before.

- Choose N (if possible) st any non-trivial elmt sends any edge adjacent to v_P to an edge making a large angle.

(we choose specifically bits of P which have large angle on v_P.)



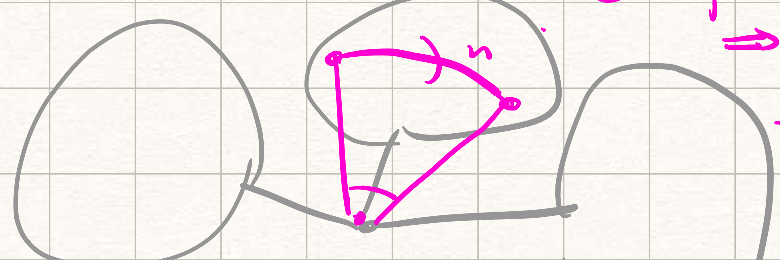
Blue edges connect v_P to a P-orbit in the horosphere $\mathcal{H}_P$.

Main claim: Let $\dot{X} = (X \setminus \underbrace{H}_{\text{remove horoball}}) \cup \underbrace{\{v_P, \ell(p)\}}_{\text{coned off horosphere}}$

Then $\dot{X} \to {}_{\langle\langle N \rangle\rangle}\backslash \dot{X}$ is a local isom away from the vertices v_P.

Indeed, large rot takes convex subset ↦ disj. convex subset.

Assume NOT. Then:



path avoiding cone pt. ⇒ angle is bounded. ⇒ The angle not large enough. (by choice of N).

□

Facts:

"$N \gg 1$" $\dot{X}$ is locally Gromov. hyperbolic.

"$N \gg 1$" $\dot{X}$ (coarsely) simply connected.

Thm: (Cartan-Hadamard-Gromov)
If a space is simply connected,
$10^7\delta$-locally δ-hyp, then it is
300δ-hyp.

want spaces up to size $10^7\delta$ to be hyp

$\Rightarrow$ so want vertices to be $10^7\delta$ apart

$\Rightarrow$ want to take "deep" horoballs so that verts at least that apart.

$\Rightarrow$ large rotation
= for which $n \in \mathbb{N}$ st the argument holds.

$\Rightarrow$ put convex subset into disj convex subset

then $\mathcal{F}$ = all elements need to avoid st N does that.