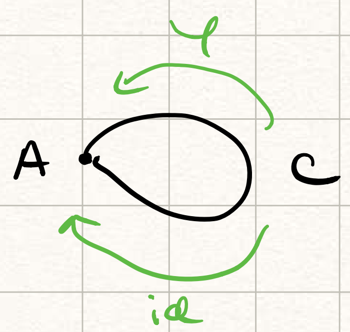
Refs: Bogopolski

- Topological Grp Th. CTC Wall, Peter Scott

other case: $\pi_1(G\backslash T) \neq 1$

$\rightsquigarrow$ HNN extension

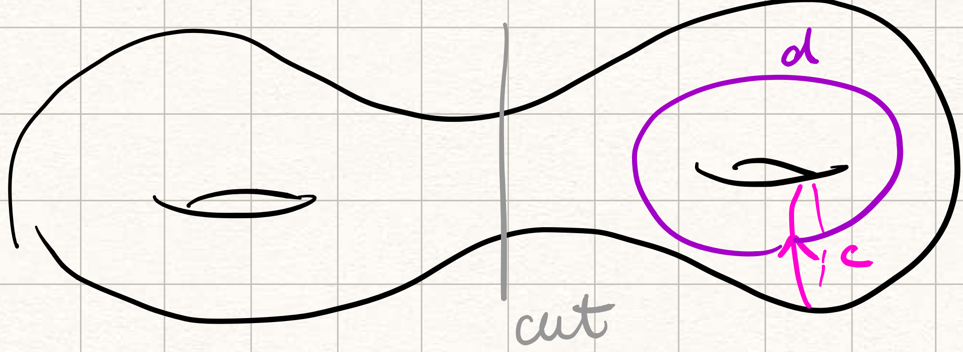


The HNN ext. corr to setup $A, C \leq A$ and maps

$id : C \hookrightarrow A$

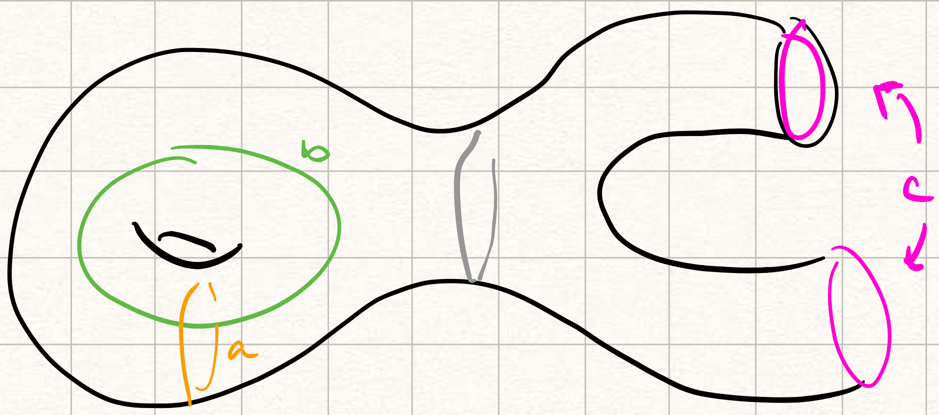
$\varphi : C \longrightarrow A$

$$HNN(A, C) = \langle A, t \mid t \overbrace{\psi(c)}^{\psi = 1} t^{-1} = \varphi(c) \quad \forall\, c \in C \rangle$$



Cut along a NON-separating curve

$\langle F(a,b,c), d \mid [a,b][c,d] \rangle$

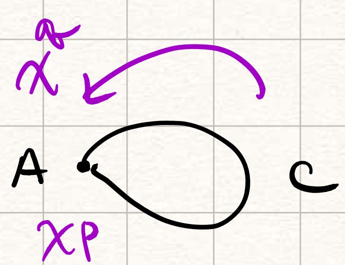


(check labels)

$\langle x \rangle$

$BS(p,q) = \langle x, \underline{t} \mid t x^p t^{-1} = x^q \rangle$

stable letter.



$BS(1,n) \cong \mathbb{Z}[1/n] \rtimes \mathbb{Z}$ solvable

Right angled artin grps

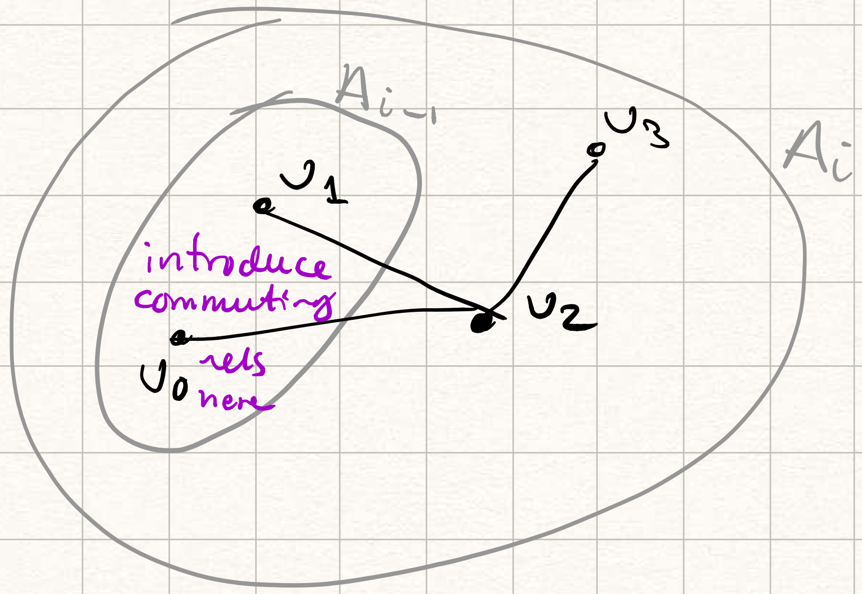
Γ graph, $\Gamma = (V, E)$

$A_\Gamma = \langle V \mid [u,v] = 1 \ \forall (u,v) \in E \rangle$

choose ordering for $\{v_0, \ldots, v_n\}$

$$A_i = \langle A_{i-1}, v_i \mid v_i a v_i^{-1} = a \quad \forall a \in A$$

A subgraph



ex:

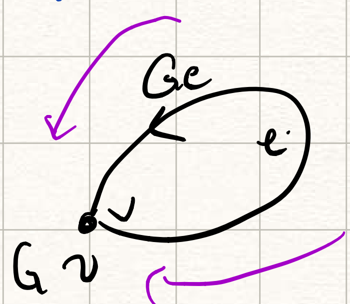
$$H_4 = \langle a, b, c, d \mid aba^{-1} = b^2,\ bcb^{-1} = c^2,\ cdc^{-1} = d^2,\ dad^{-1} = a^2 \rangle$$

$$\cong \Big(\overset{BS(1,2)}{\langle a, b \rangle} \underset{\langle a \rangle}{*} \overset{BS(1,2)}{\langle d, a \rangle} \Big) \underset{\substack{\langle b, d \rangle \\ \cong \\ F_2}}{*} \Big(\overset{BS(1,2)}{\langle b, c \rangle} \underset{\langle c \rangle}{*} \overset{BS(1,2)}{\langle c, d \rangle} \Big)$$

Q: What does the tree for an HNN-ext look like?

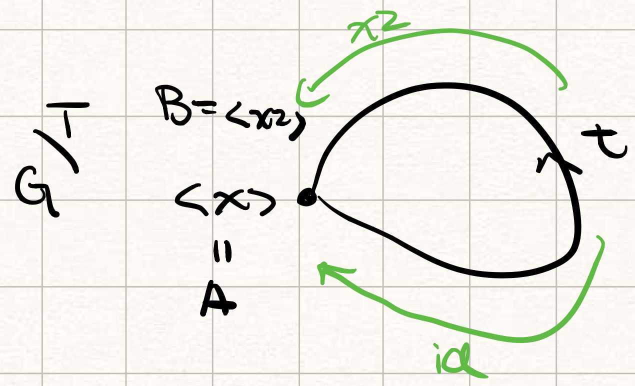
$V =$ cosets of G_v

$E =$ cosets of G_e



$B(1,2) = \langle x, t \mid t x t^{-1} = x^2 \rangle$

<u>Recall:</u>

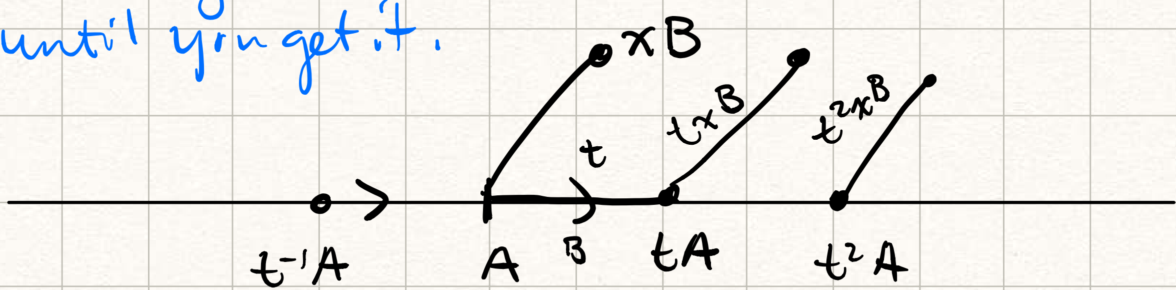


(incoming outgoing in univ cover)

$\downarrow$ univ cover

Ask how to do this again until you get it.

$B = \langle x^2 \rangle$



<u>exc</u>: figure what this all means esp Valency.

Def: A graph of grps (G, Γ) is the following set of data.

$\Gamma = (V, E)$ is a graph

for each $v \in V$, we have a grp G_v,
for each $e \in E$, we have a grp G_e,

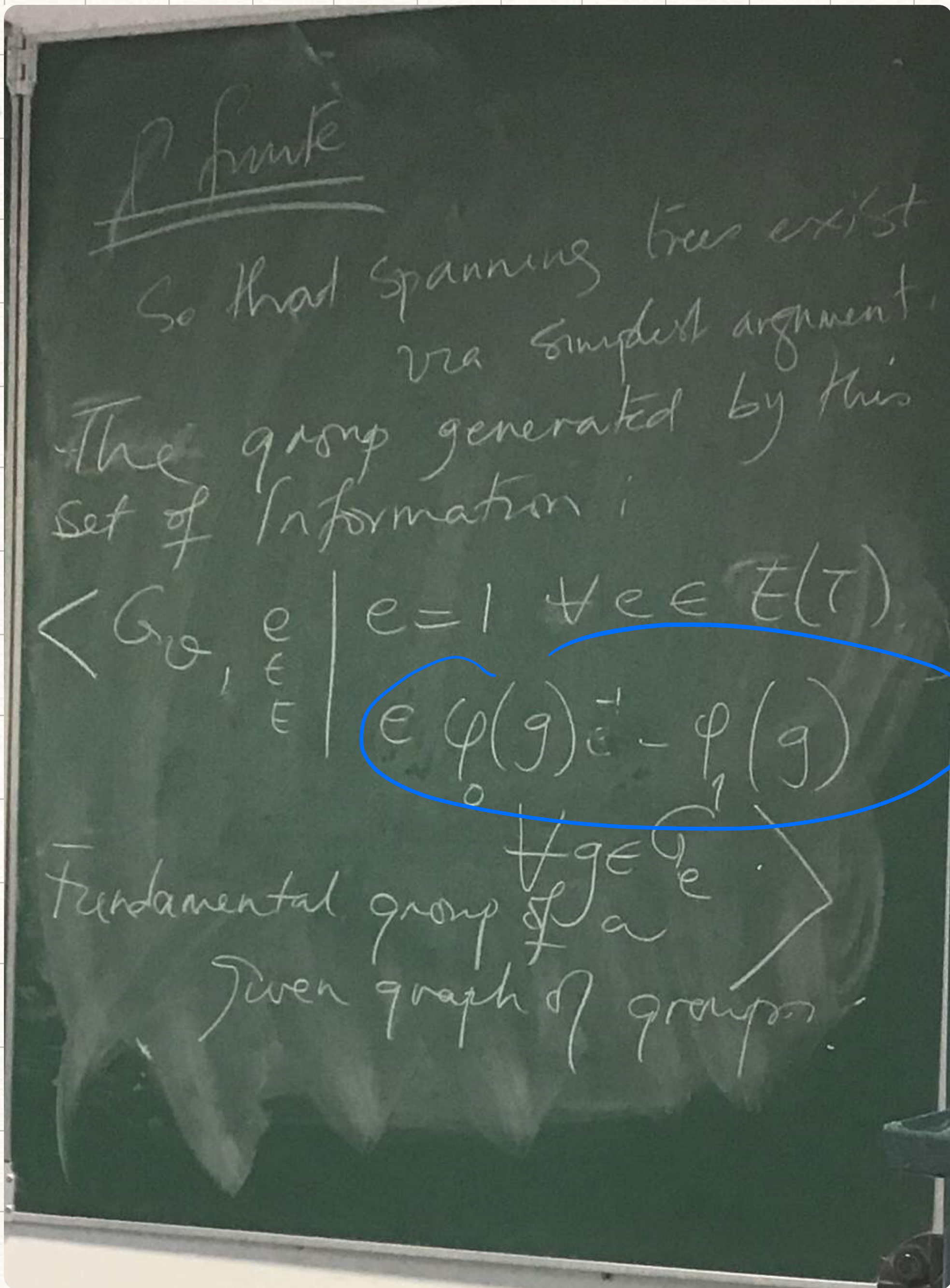
and maps $\varphi_o : G_e \to G_{o(e)}$
$\varphi_t : G_e \to G_{t(e)}$

o = origin of edge
t = terminus of edge

ex: Γ finite $\rightarrow$ so that spanning trees exist via simplest argument.

The grp generated by this set of information

$$\left\langle G_v, \underset{\substack{\cap \\ E}}{e} \;\middle|\; \begin{array}{l} e = 1 \;\forall e \in E(T) \\ e\varphi_o(g)e^{-1} \to \varphi_1(g) \;\forall g \in G_e \end{array} \right\rangle$$



? 2

Thm: $(G, \Gamma) \rightsquigarrow$ Action on a tree

Get $(G, \Gamma) \leftsquigarrow$ G acts on a tree T simplicially

$\Gamma = G \backslash T$ grp G_v

we have a grp G_e

and maps: $\left.\begin{array}{l} \varphi_0 : G_e \longrightarrow G_{o(e)} \\ \varphi_t : G_e \longrightarrow G_{t(e)} \end{array}\right\}$ injective maps.