

Disk diagram

planar sq. complex, contractible

$$D \rightarrow X$$

(not nec. CAT(0))



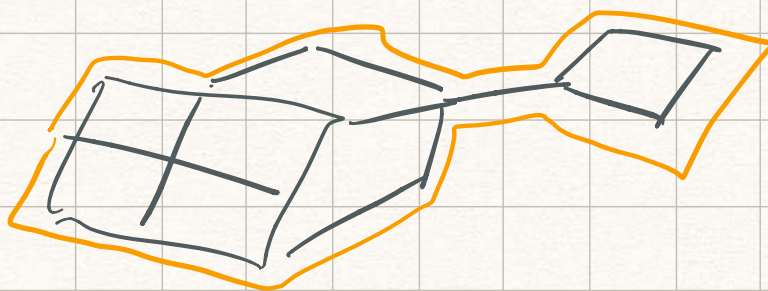
Pf: $x, y, z \in X^{(0)}$

$$\Delta(x, y, z)$$

X is 1-connect, ie,

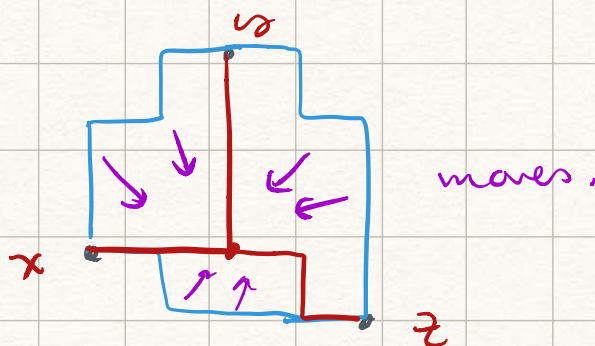
$$\exists D \rightarrow X : \partial D \rightarrow \Delta(x, y, z)$$

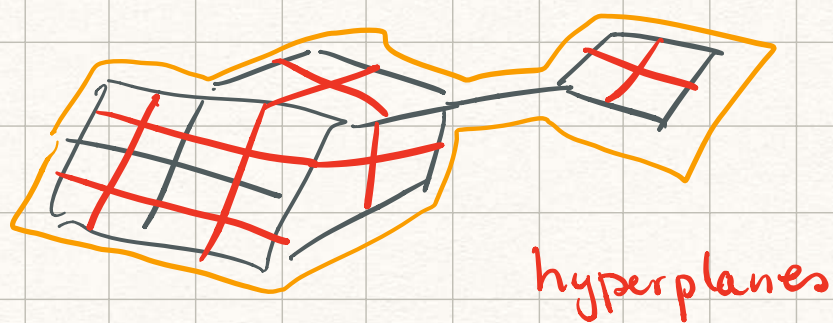
disk diagram



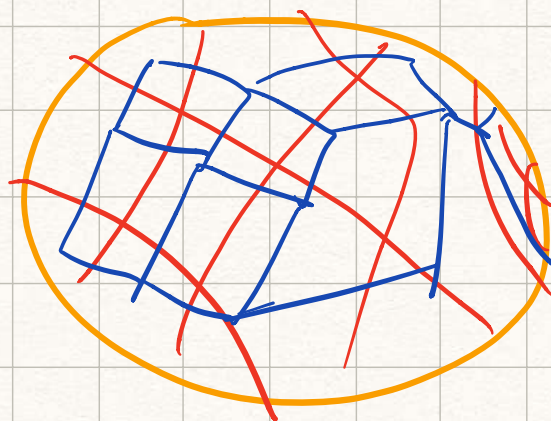
idea: Change D using the following moves,
until $\Delta(x, y, z)$ a tripod

ex:

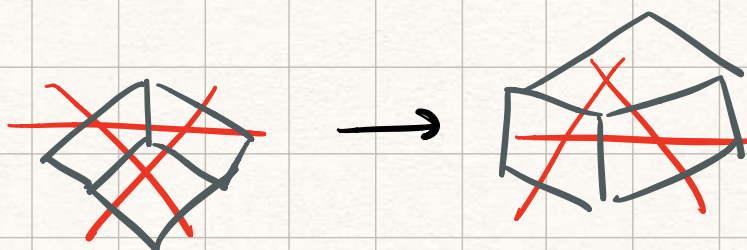
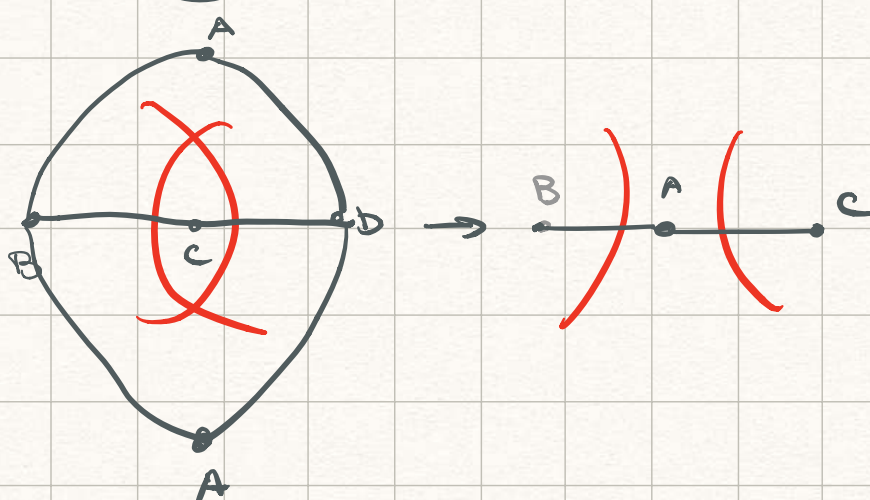
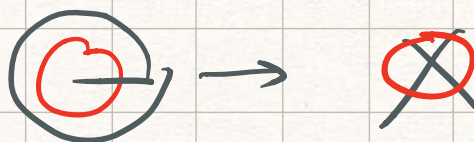




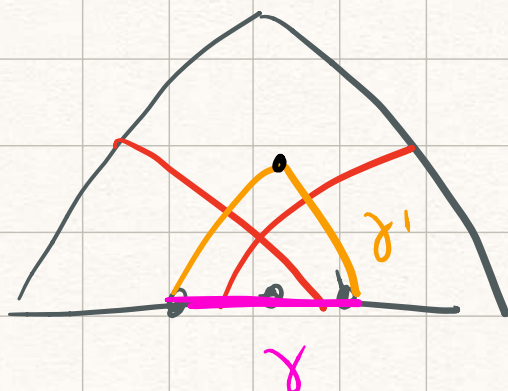
deal
= the sqs



moves



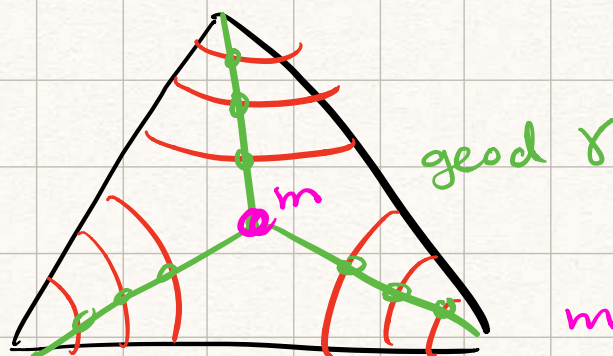
PT: you can uncross hyperplanes in Δ .



γ, γ' geod

γ' remains
1 intersect.

$\Rightarrow \dots$



geod γ

m is unique
(exc)

Helly property

if $C_1, \dots, C_n$ are convex subcplx

st $C_i \cap C_j \neq \emptyset \quad \forall i, j$

$\Rightarrow \bigcap_{i=1}^n C_i \neq \emptyset$

Pf: Suffice to show for C_1, C_2, C_3 .

set $x_{ij} \in C_i \cap C_j$

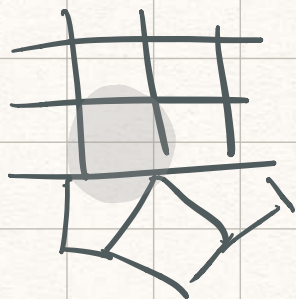
$\Rightarrow m(x_{12}, x_{13}, x_{23}) \in \bigcap C_i$

$\underbrace{\quad}_{C_1} \quad \underbrace{\quad}_{C_2}$
 $\underbrace{\quad}_{C_3 \dots}$

From median graph to CCC

Given Λ median graph define

$X_\Lambda = \Lambda$ w/ a cube glued
in to every embedded
 Δ -skeleton of a cube.



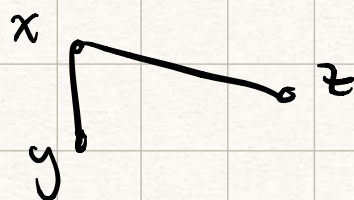
Thm: X_Λ is CCC.

Pf: Simple connect,

Obs: $\overset{x}{\curvearrowright} \overset{y}{\curvearrowright}$

$$\forall z \in \Lambda \quad |d(x, y) - d(y, z)| = 1$$

$\Rightarrow$ no $\triangle$



Obs: No $K_{3,2}$.

Now since CCL $\rightarrow$ graph,
want to define Half-sp:

$$H_{xy} = \{z \mid d(z, y) + 1 = d(z, x)\}$$

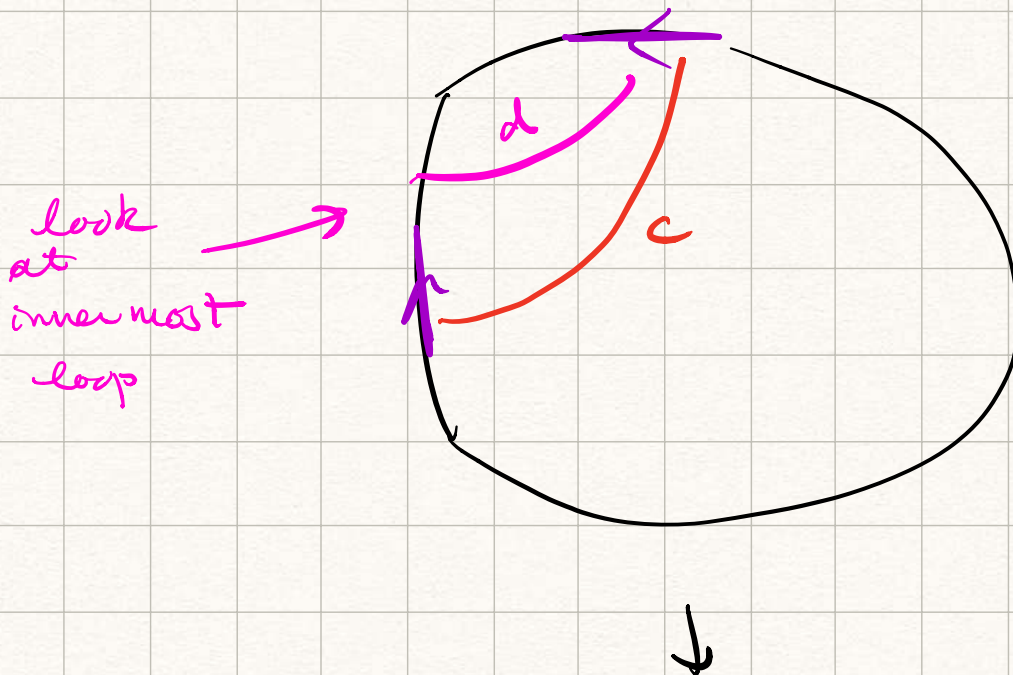
$$\Leftrightarrow d(z, y) < d(z, x)$$

ie: $H_{xy} \sqcup H_{yx} = \wedge$



you are either
close to x or
close to y .

let c be the shortest loop in $X \setminus \wedge$
which is NOT trivial



Pocsets (poset + c)

X is ccc

$\mathcal{H}(X) = \text{half sp of } X$

$h \leq k$

$h^* \leftarrow \text{the other half sp}$

A Pocset $(P, \leq, *)$

is a poset $(P, \leq)$

w/ $*$: $P \rightarrow P$ order reversing involution

st h, h^* are incomparable

$\forall h \in P.$

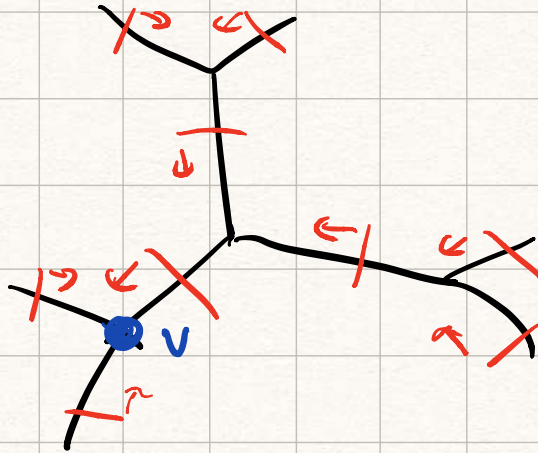
- $(P, \leq, *)$ is locally finite if
 $\forall h \leq k, \# \{l \mid h \leq l \leq k\} < \infty$

- h, k are transverse if any two
of h, h^*, k, k^* are incomparable

- P has finite width if $\exists n$ st
 $h_1, \dots, h_m$ are pw transverse
 $\Rightarrow m \leq n.$

X

Goal : pocset $\rightarrow$ median graph $\rightarrow$ ccc.



look at all $\frac{1}{2}$ -sp which has v .
only 1 side pt to v .