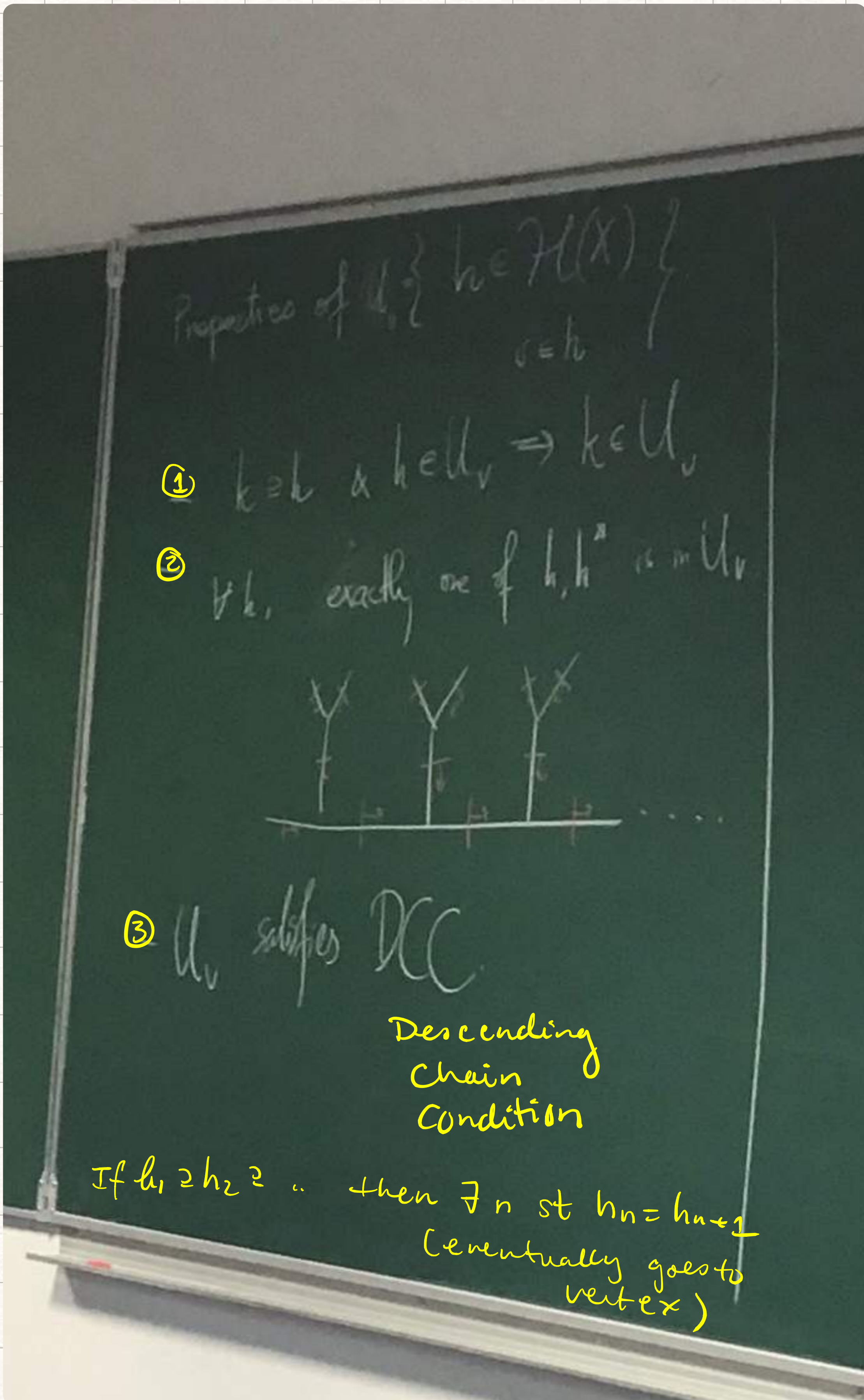



ultrafilter

Descending Chain Condition

If $h_1 \supseteq h_2 \supseteq \ldots$ then $\exists n$ st $h_n = h_{n+1}$ (eventually goes to vertex)

# From Pocsets to CCC

↑ locally finite, has finite width. bound on half-space & complements.

- Define the vertices of CCC $X$ to be all DCC ultrafilters.


$\iff |v \,\Delta\, w| = 2$

$\Updownarrow$

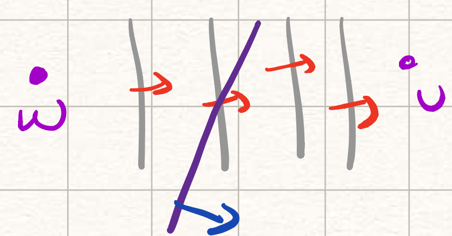
$\exists!\, h : h \in v$
$h^* \in w$.

$\Rightarrow$ A graph $X^{(1)}$

ex: $X^{(1)}$ is connected

The distance formula holds.

$$d(v,w) = \#\{h \in v \mid h^* \in w\}$$




$X^{(1)}$ is a median graph:

$m(x,y,z) = \{h \in P \mid h$ is in at least 2 of $x, y, z\}$

check DCC + prove it's median.

$\rightsquigarrow X^{(1)}$ median graph $\rightsquigarrow X$ is CCC $\square$

# Sageev-Roller Duality

$$\left\{\begin{matrix}\text{finite}\\ \text{dim}\\ \text{CCC}\end{matrix}\right\} \longleftrightarrow \left\{\begin{matrix}\text{Posets}\\ \text{w/ finite}\\ \text{width \&}\\ \text{loc. finite}\end{matrix}\right\}$$

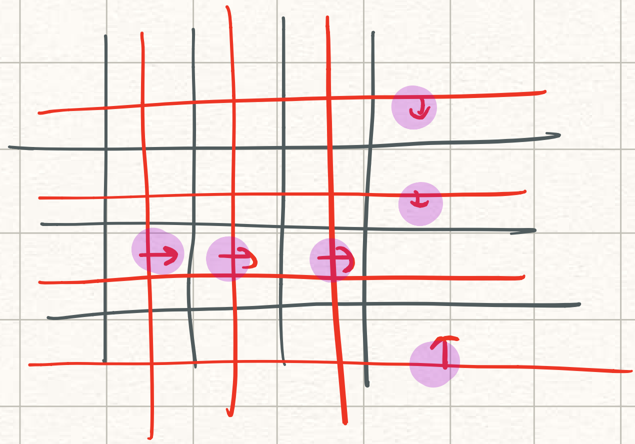
$$X \longrightarrow \mathcal{P} = (\mathcal{H}, \leq, *)$$
$$X_{\mathcal{P}} \longleftarrow \mathcal{P}$$

## The Roller boundary

The Roller ~~boundary~~ compactification of a CCC (or Poset) is the set of all ultrafilters

$$\subseteq \prod_{h \in H} \{h, h^*\}$$

$\uparrow$ compact.

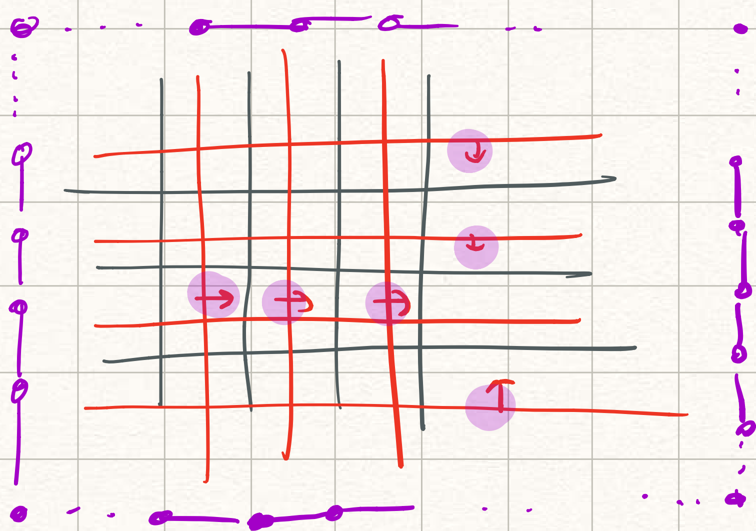


converges to this pt at $\infty$,

$p$ •

$p \notin$ median graph,
but is a pt in boundary.

continue:



# New constructions of CCC

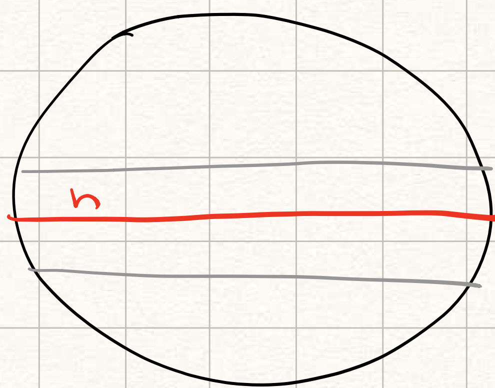
1. 

$$\begin{array}{ll} X & \text{ccc} \\ \downarrow & \\ (\mathcal{H}, \le, *) & \\ \rotatebox[origin=c]{90}{$\subseteq$} & \\ (\mathcal{H}', \le, *) & \longrightarrow \quad Y \ \text{ccc} \end{array}$$

contraction map (by restricting map to $\mathcal{H}'$).

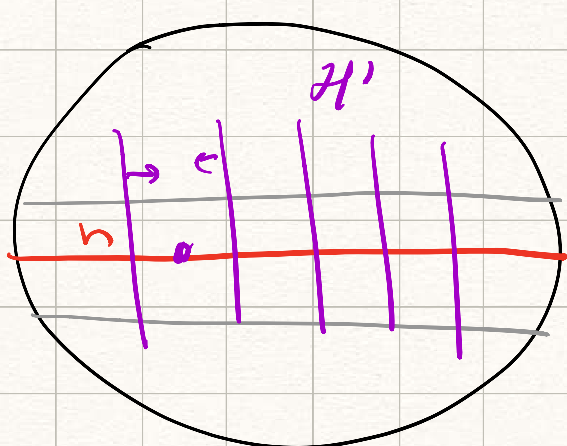
cubical construction.

Q:




What is the poset struct of h?

A: The vert are defined by hyperplanes pointing to it:




etc.

$$\begin{array}{lcc} 2. & (P, \le, *) & , \quad (P', \le, *) \\ & \rightsquigarrow & \rightsquigarrow \\ & X & X' \end{array}$$

$$(P \sqcup P', \le, *) \rightsquigarrow X \times X'$$

3. foldings, resolution.

ex:

but sometimes get uninteresting things

1) $\mathbb{Z}^2 \curvearrowright \mathbb{R}^2$

(Rmk $G \curvearrowright P(H, \le, *)$
$\rightsquigarrow$
$G \curvearrowright X$

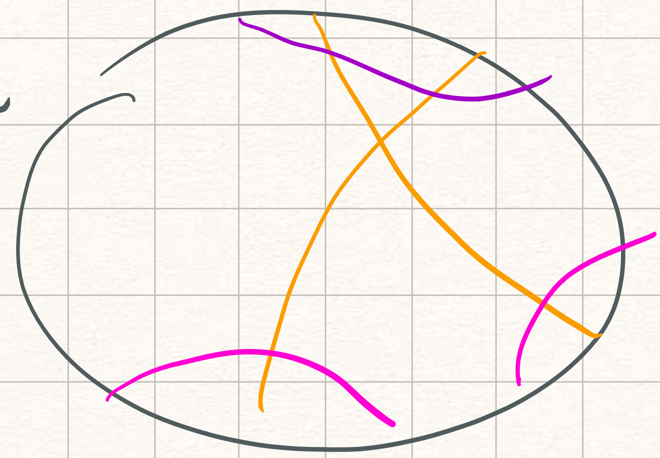
$\mathcal{L} =$ 

$\mathbb{R}^2 \setminus \mathcal{L}$ each line defines 2 half.sp $\cap\ \mathbb{R}^2 \setminus \mathcal{L}$

exc: Show $X_{\mathcal{L}} = \mathbb{R}^3$

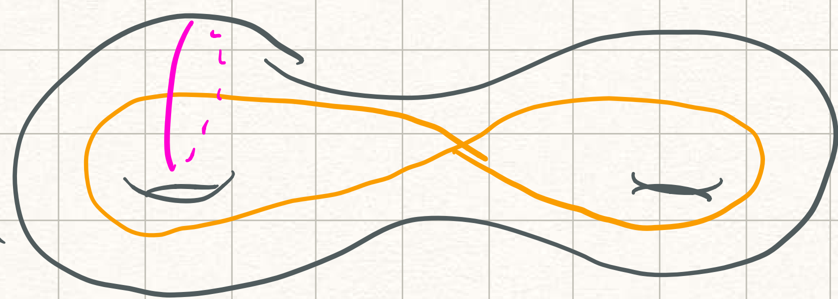
MEH!

2) $G \curvearrowright$



$\mathbb{H}^2 = h^2$

$\downarrow$

$G = \pi_1\Big($  $\Big)$

$G \curvearrowright X_2$ is co-cmpt, b/c

x every collection of pairwise (pw) intersecting line pass through some ball $B_R$.

dual = a cube!