Notes : on HIM page   Hey these might be good :) !

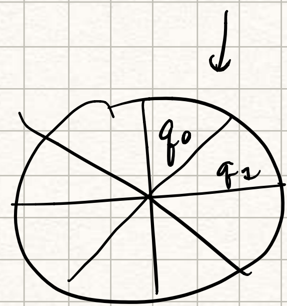
Computing

Def (7.1 & 7.2) [Turing Machine]

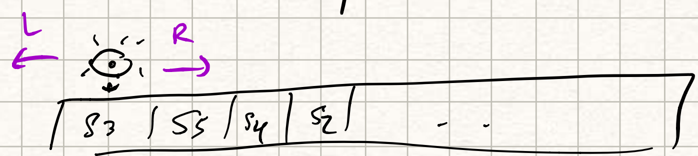
- Finite alph. $S = \{s_0, \ldots, s_n\}$
- States $Q = \{q_0, q_1, \ldots, q_m\}$

  q_0 start

  q_2 halt



- Machine has tape



read

move Left or Right

has transition funct.

Quadruple $(\underbrace{q_i, s_j}, \; s_k \cup \{L, R\}, \; q_\ell)$

takes state q_i w/ letter s_j

replace s_j it w/ s_k

move to state q_ℓ

- If we ever reach state q_0,

  STOP & output tape.

Def (7.3 & 7.4)

If T eventually halts on input $w \in S^*$

write $T(w)\downarrow$

else $T(w)\uparrow$.

Def (The halting set of T)

$$\Omega(T) := \{ w \in S^* \mid T(w)\downarrow \}$$

Def (7.5)

- Say $A \subseteq S^*$ is recursively enumerable if $A = \Omega(T)$ for some T.
- Recursive if both A & $S \setminus A$ are recursively enum. (r.e).

" Recursive $\Leftrightarrow$ Computable "

R.e: can check for membership

$\hookrightarrow$ wait for it to halt (or NOT)

Recursive: can check for

- membership
- NOT membership (built T' st $S \setminus A = \Omega(T')$)

Church-Turing thesis

General computation $\equiv$ Turing Machine

$\left\{ \begin{array}{l} \text{finite input} \\ \text{finite \# steps} \\ \text{finite output} \end{array} \right\}$

$\cup$ I

computers

$S_0, S_1, \ldots$ quads ,

$q_0, q_2 \ldots$,

Can give a canonical numbering of Turing M's

$T_1, T_2, \ldots$

<u>Def</u> (7.8 , 7.9)

"<u>The</u>" Halting Set $\mathbb{K}$ as:

$$\mathbb{K} := \{ n \in \mathbb{N} \mid T_n(n) \downarrow \}$$

Can express a word in alphabet as some integer on base $|S|$.

(ex : $S_1, \ldots, S_6$ $\to$ base 6).

- <u>$\mathbb{K}$ is r.e.</u> meaning it's equal to the Halting set of some turing machine which somehow accesses some info on other TM's (?)

· **But** $\mathbb{K}$ is NOT recursive.

$(\mathbb{N} \setminus \mathbb{K} \neq \Omega(T'))$

Next: Build TM in grp st operation = mult.

r.e. $\Rightarrow$ nice enough to put into re grp.

NON-RC $\Rightarrow$ that grp might have some incomputable problem in it.

Q: $\mathbb{K}$ is NOT Recursive b/c it self-references...

if $\mathbb{N} \setminus \mathbb{K}$ is r.e.,

$\Rightarrow \mathbb{N} \setminus \mathbb{K} = \Omega(T_m)$

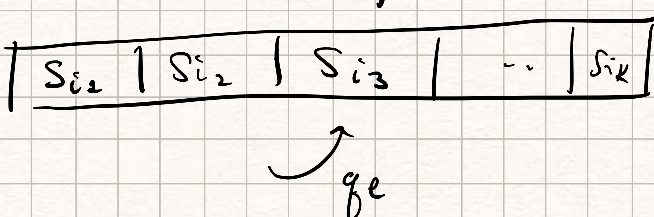
$m \in \mathbb{K} \iff T_m(m)\downarrow$

$\iff m \in \Omega(T_m)$

$\iff m \in \mathbb{N} \setminus \mathbb{K}$.

Def: (8.10)

An instantaneous description of a Turing M is given by



$\rightsquigarrow s_{i_1} s_{i_2} q_\ell s_{i_3} \cdots s_{i_k} \in (S \cup Q)^*$

Point: Snapshot of TM at a certain time.

Idea: Hard to realize an instantaneous description in an fp grp (combinatorics on words).

it is easier: to realize integers in a grp (power of elts)

Def: (9.10) A **modular machine** (MM) is:

i) A modulus $m > 1$ & $\in \mathbb{N}$

ii) A finite set of quadruples of form

(a, b, c, R) or (a, b, c, L) w/
'right 'left

$m > a, b \geq 0$, $m^2 > c \geq 0$

St for each pair (a, b) $\exists$ at most one such quad. of form $(a, b, *, *)$

NOTE: TM = exactly one pair of quad
MM = at most one pair.

Def

A MM config. is an ordered pair $(\gamma,\beta)\in\mathbb{N}^2$.

Write $(\alpha,\beta) \xrightarrow[M]{} (\alpha',\beta')$ called a computational step of M if:

$$\alpha = um+a,\ \beta = vm+b \quad (0 \leq a,b,c)$$

& $\exists$ c st

1) $(a,b,c,R) \in M$ & $\alpha' = um^2+c,\ \beta' = v$

OR

2) $(a,b,c,L) \in M$ & $\alpha' = u,\ \beta' = vm^2+c$

R $\quad um^2 + \cancel{a}c, \quad um^2 + \cancel{b} + c$

L

Idea: simulate walking on a tape.

NB: The action of M on (α,β) depends only on the class of (α,β) mod m.

Write: $(\alpha,\beta) \xrightarrow[M]{*} (\alpha',\beta')$

if $\exists$ finite seq

$$(\alpha,\beta) \xrightarrow[M]{} (\alpha_1,\beta_1) \to \cdots \to (\alpha',\beta')$$

Such a seq is a computation of M.

If, for $\alpha = um + a$, $\beta = vm + b$, $0 \leq a, b \leq m$,

NO quad. of M begins $(a, b, *, *)$ then

(α, β) is terminal in M.

If $(0, c)$ is terminal in M, write

$$H_0(M) = \{ (\alpha, \beta) \mid (\alpha, \beta) \xrightarrow[M]{*} (0, c) \}$$

"Halting set" of sorts.

How do you interpret an inst. descript of a TM as a pair of ints?

Let T be the alphabet S, states Q.

Set $m := |S| + |Q|$.

Rewrite $S = \{0, \dots, n\}$

$Q = \{n+1, \dots, m-1\}$

Let $\underbrace{b_k b_{k-1}, \dots, b_1 b_0}_{u}\, q\, a\, \underbrace{c_0 c_1 \dots c_l}_{v}$

be an inst. descript of T.

Set $u = \sum_{i=0}^{k} b_i m^i$

$v = \sum_{i=0}^{\ell} c_i m^i$

MM → encode this in ints. two directions on the tape

(α, β), (α', β')

$\leftarrow$ $\rightarrow$

depending on dir.

To this inst. descript, we associate 2 MM configs.

$$\underbrace{\underbrace{b_k\, b_{k-1}, \dots, b_1\, b_0}_{u},\ q}_{\text{integer 1}}\ \underbrace{a\ \underbrace{c_0\, c_1 \dots c_\ell}_{v}}_{\text{int 2}}$$

1) $(um+a,\ um+q)$ (left ass.)

2) $(um+q,\ um+a)$ (right ")

Thm (9.2, 9.3)

Let T be TM. Then from v+(?) we can construct a MM "M" st if $w \in S^*$ in T, & (α, β) the left associatives of q, w in M. Then $T(w)\downarrow \iff (\alpha, \beta) \in H_0(M)$