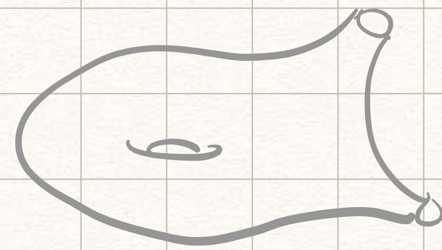
Goal: $S = S_{g,n}$



$MCG(S) = \mathrm{Diff}(S) / \mathrm{Diff}^0(S)$

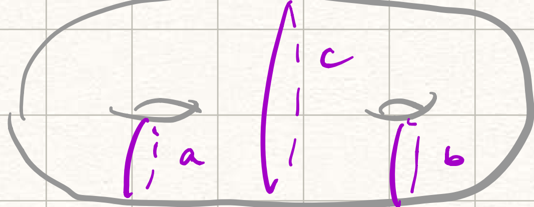
Coarse struct — hyp.
— flat.

I – "In the surface":
- curves & laminations
- curve complex
- subsurface proj.

II – "In the grp".

What's in MCG?
- Abelian subgrps

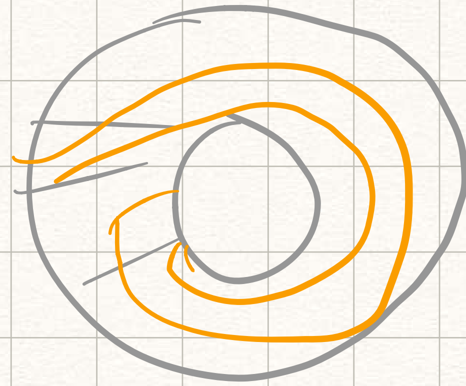
T_a = Dehn twist



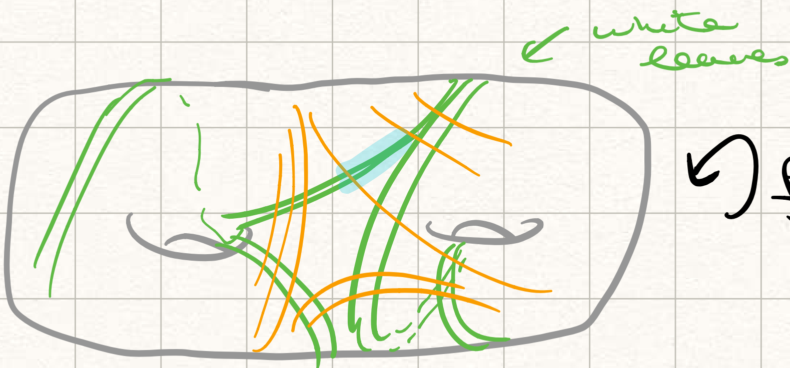
$T_a T_b = T_b T_a$.

$\langle T_a, T_b \rangle \cong \mathbb{Z}^2$

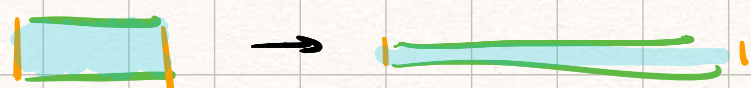
Dehn twist:



"Hyperbolic elmts": pseudo-Anosov



$\circlearrowleft f$



λ_+, λ_- transverse (measured) lamination.

f stretches λ_+

shrinks λ_-

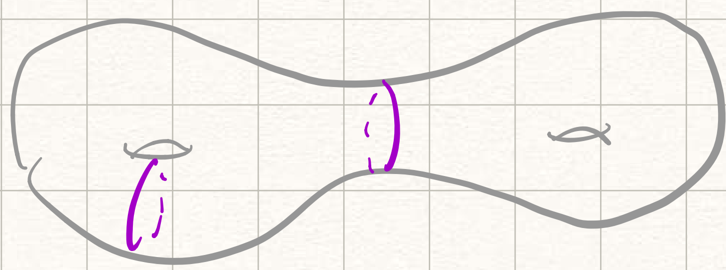
f pseudo-Anosov

$$\Rightarrow C(\langle f \rangle) = \{g : gf = fg\}$$

$$\vee \text{ f.i.}$$

$$\langle f \rangle$$

- Reducible f preserves a multicurve.



- finite order: $f^n \sim id$

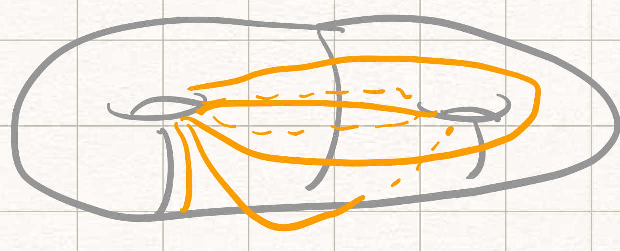
Thm (Thurston)
Every elmt in $MCG(S)$ is of these types.

Thm (Birman-Lubotzky-McCarthy)
A subgrp of $MCG(S)$ is virt. ab. or contains F_2.

flat struct

hyp.

f, g pseudo-A w/ different laminations, then $\exists$ power m st $\langle f^m, g^m \rangle = F_2$.

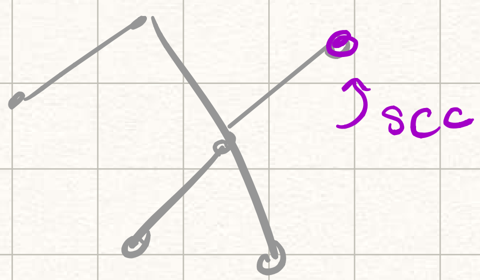


Def: (Harvey)

The complex of curves $\mathcal{C}(S)$

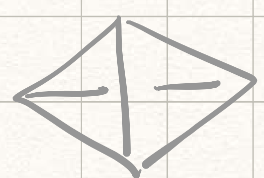
has: simplex $\longleftrightarrow$ multicurve

face $\longleftrightarrow$ inclusion



(essential) curve : not ∂(disk)

not parallel to ∂S



t-simplex

$\updownarrow$

$\mathbb{Z}^t$

- $\dim \mathcal{C}(S) < \infty$

(A) · locally infinite

Thm: (Mazur + Minsky)
$\mathcal{C}(S)$ is δ-hyp.

Fact: $\operatorname{diam} \mathcal{C}(S) = \infty$

Q: $\partial \mathcal{C}(S) = ?$

$\chi(S) < 0 \Rightarrow S$ admits a hyp. metric.

Def: A geod lamination is a closed subset of S foliated by geodesic.

locally, looks like:

$\Big\}$ $[0,1] \times K$
K compact.

Facts: $\exists$ geod laminations which are minimal but have uncountably many leaves.

every leaf is dense

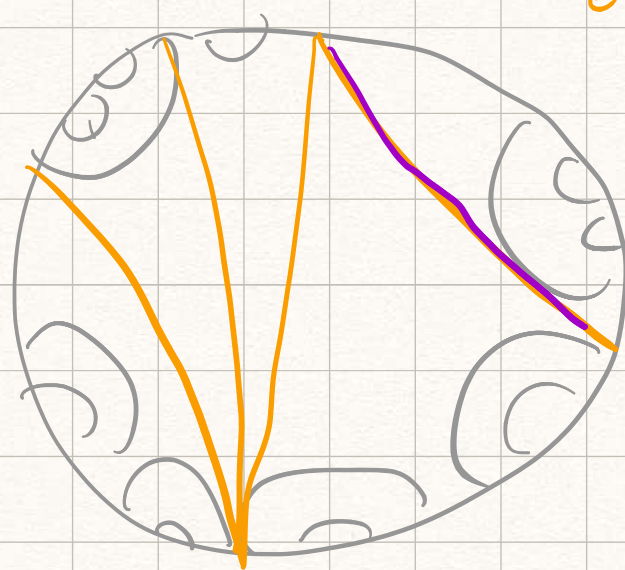
- the lams $\lambda_{\pm}$ of a pseudo-A are of these type.

Non-ex :

? still confused by ex

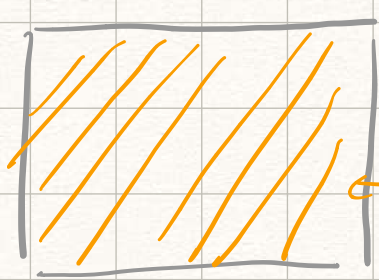
Take this geod which spirals around another closed loop

lifts to geod



PT Not SCC & not minimal

ex :

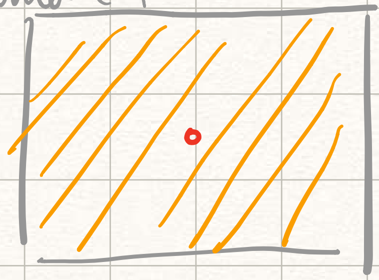


T^2.

slope of $s \notin \mathbb{Q}$

$\Rightarrow$ leaf is dense.

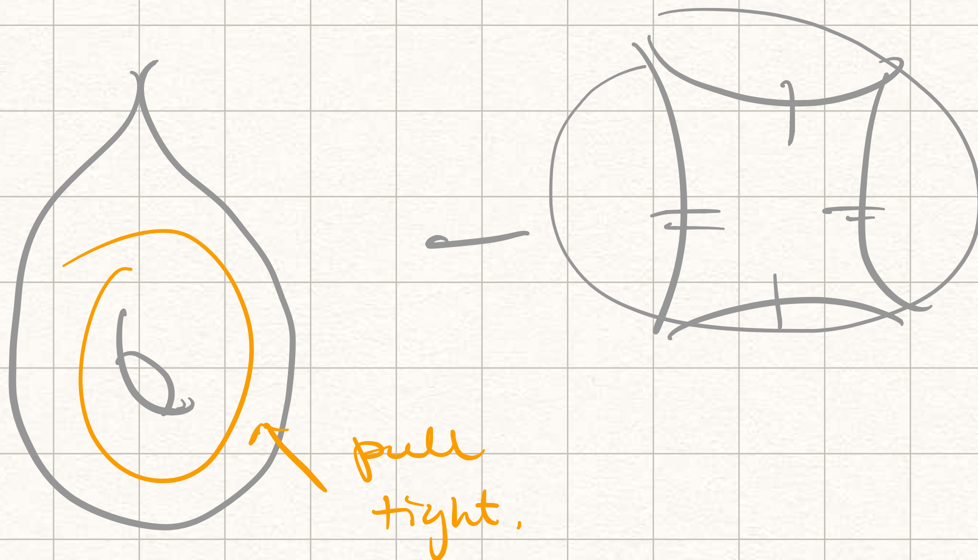
Now remove pt: $\Rightarrow$ χ euler char < 0



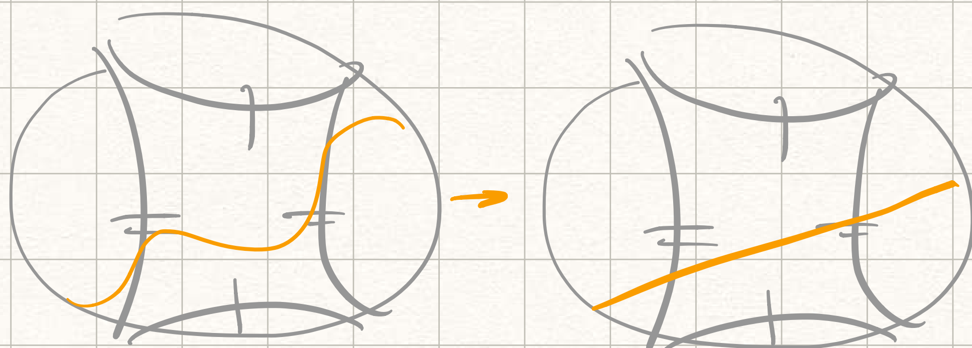
$T^2 - \{x\}$

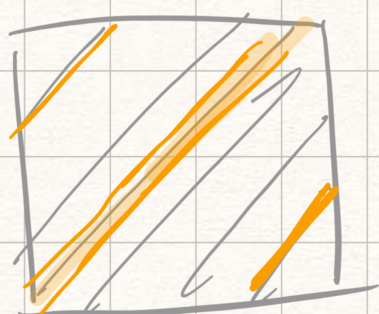
$\downarrow$

By pulling tight, the // which was geod in $\square$ will become geod in $\boxed{\cdot}$



ie

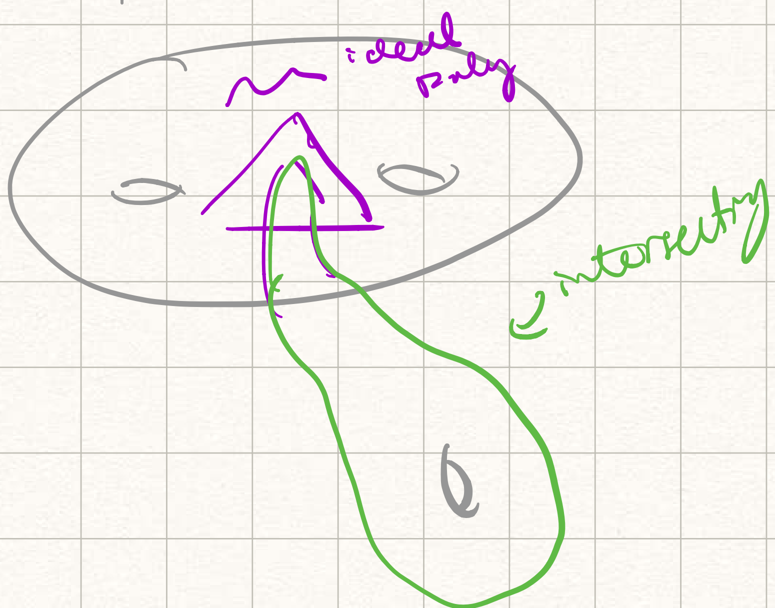


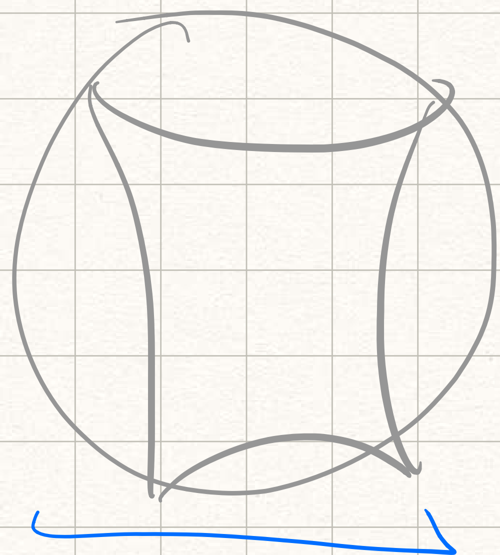


$\Rightarrow$ is a dense lamination.

A: Why is $\mathcal{C}(S)$ of ∞-diam?

Let λ be a minimal **filling** **lamination** (S w/o puncture)
(ie, $S \setminus \lambda$ is union of ideal polygons $\Longrightarrow$ every closed essential curve intersects λ.)





idk what this was for.

Lemma: If $\gamma_1, \gamma_2, \gamma_3, \ldots$ distinct sequences of simple closed geods converging to a lamination, $\hat{\lambda} \supset \lambda$, then $\mathrm{dist}_{\mathcal{C}}(\gamma_1, \gamma_n) \to \infty$

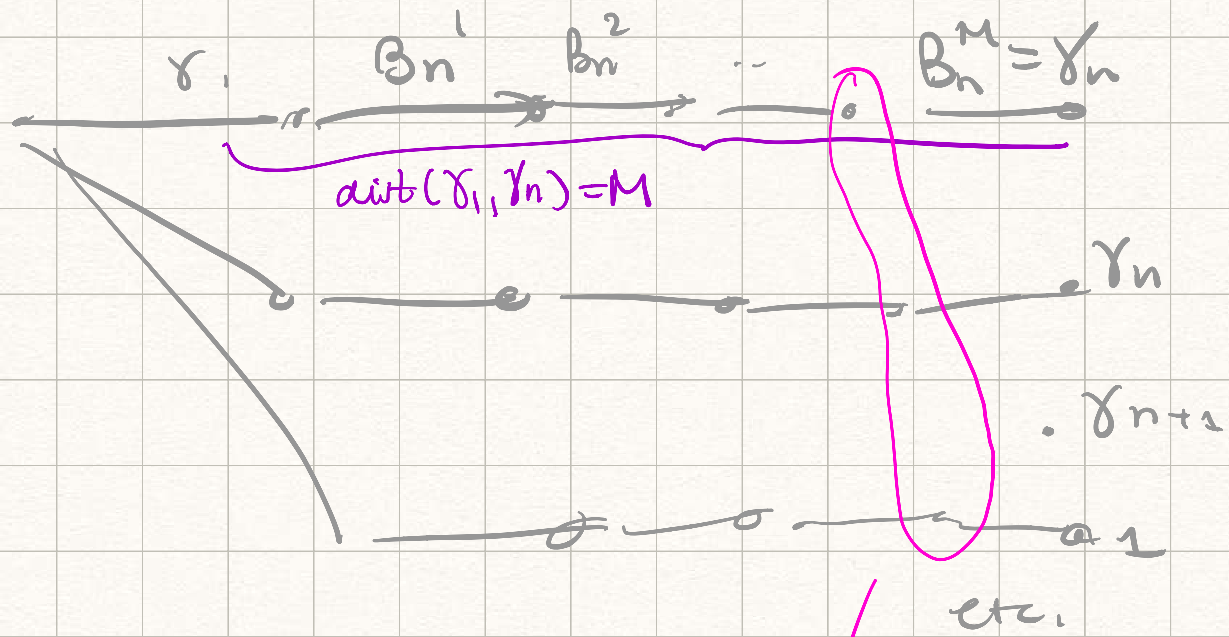
the dist in curve complex goes to ∞
(ie, they intersect a lot)

Pf: Suppose $\exists\ \gamma_1, \gamma_2, \ldots$

$d_{\mathcal{C}}(\gamma_1, \gamma_n) = M$

then

if dist $\not\to \infty$, then can take subseq where dist is ct.



These are geods. converging to some new lams.

$\beta_n^{M-1} \xrightarrow{\text{(subseq)}} \hat{\mu}$

$\beta_n^{M-1}, \beta_n^M = \gamma_n$ are disjoint (by curve complex construction)

$\hat{\mu}$ and $\hat{\mu}$ & $\hat{\lambda}$ can only intersect on leaves.

otw transverse intersect.

$\Rightarrow$ pairs of geod β_n^i, β_n^{i+1} getting longer & longer dist from each other.

$\Rightarrow$ their limits cannot intersect.

? Don't quite get it but ok.

$\Rightarrow \underbrace{\lambda \subset \hat{\mu}}$ $\leadsto$ produces new seq $d(\gamma, \beta_n^{M-1}) = M-1$

B/c $S \setminus \lambda$ is union of polygons. so λ can't be disj from $\hat{\mu}$.

$\to$ Use induction to show that $\gamma_1 = \lambda$, contra b/c γ_1 fin. length,

λ ∞-length. $\square$

Thm (Klarreich)

$$\underbrace{\partial(\mathcal{C}(S))}_{\text{Gromov boundary}} = \left\{ \begin{array}{l} \text{minimal filling} \\ \text{laminations} \end{array} \right\}$$

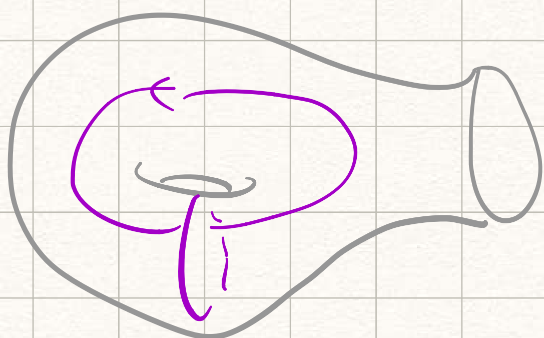
saying that the geodesics that goes to ∞ in $\mathcal{C}$ are the minimal filling laminations!

$MCG(S) \curvearrowright \mathcal{C}(S)$

Consider $\{ \mathcal{C}(W) : W \subseteq S \}$

essential sub-surfaces subspaces.

Special cases

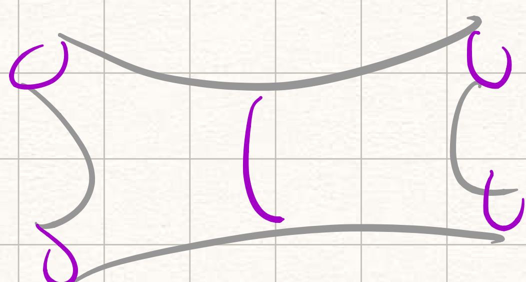


$\mathcal{C}(S_{1,1})$

any two curves intersect.

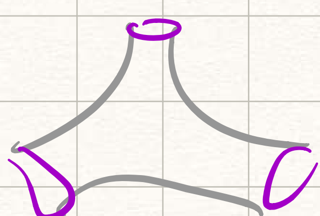
edges = curves intersecting ONCE

⇝ get Farey graph

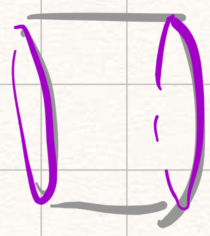


$\mathcal{C}(S_{0,4})$

Same problem.



$\emptyset$



$\simeq \mathbb{Z}$