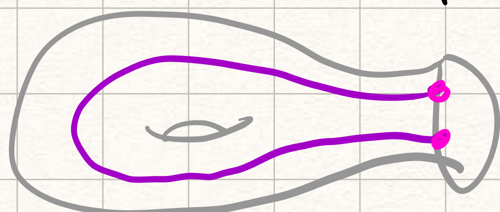
Subsurface struct & coarse struct. of MCG

- Acylindrical hyp
- Projection Complexes
- Hierarchically Hyp. grps.

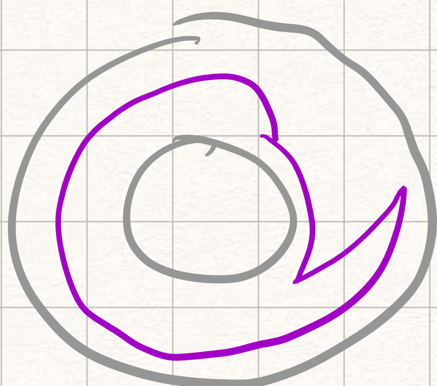
- Arc & curve complex $AC(W)$

arc = not closed geod = but endpts



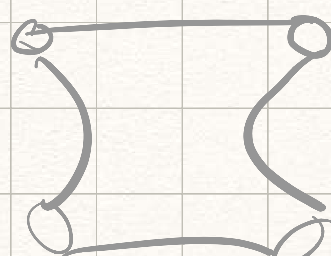
vertices curves/arcs

- $AC(W)$ qi to $\mathcal{C}(W)$



$\left.\begin{array}{l}\text{arcs/} \\ \text{homotopy} \\ \text{w/ fixed} \\ \text{endpts.}\end{array}\right\}$ qi to $\mathbb{Z}$

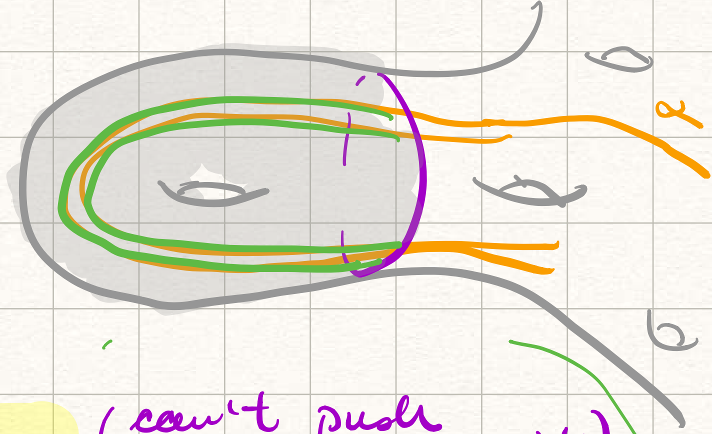
except for exceptions:



Projections

W subsurfaces

α curve in S



if $\alpha \cap W$ essentially (can't push α to be disj from W)

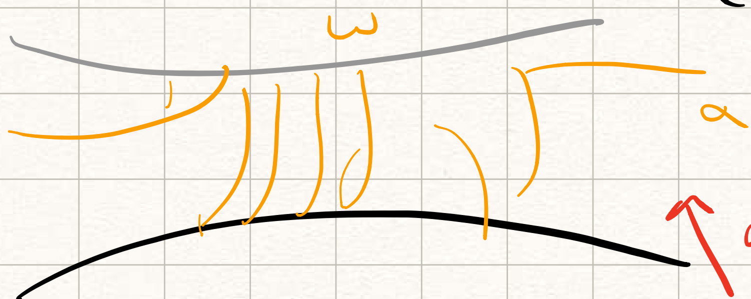
then $[\alpha \cap W]$ = simplex in $AC(W)$

these are the arcs in the simplex

this induces a map:

$\pi_W : \mathcal{C}(S) - \{\alpha \text{ disj. from } W\} \to \mathcal{C}(W)$ ← up to parallel copies.

except doesn't work for case of annulus: (but works for other cases)



α is NOT well def'ed

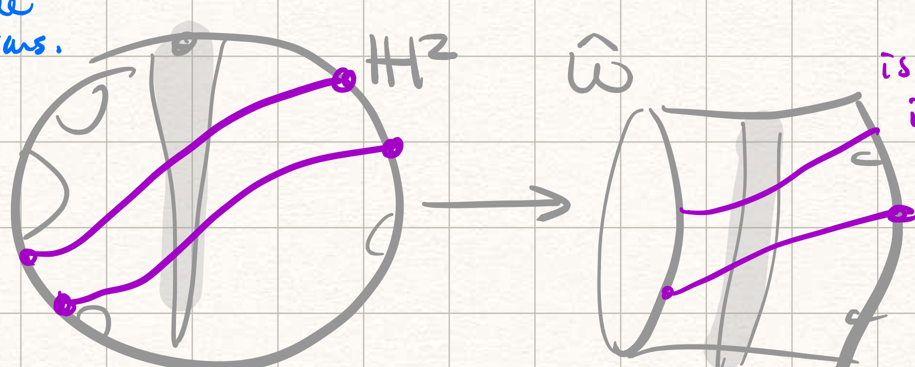
$\hat{W} = \mathbb{H}^2 / \underbrace{\langle \text{core } W \rangle}_{\gamma}$ } grp.

$\cup(\partial\mathbb{H}^2 \setminus \text{fix}(\gamma)) / \langle\gamma\rangle$

Need to go to univ cover $\hat{W}$

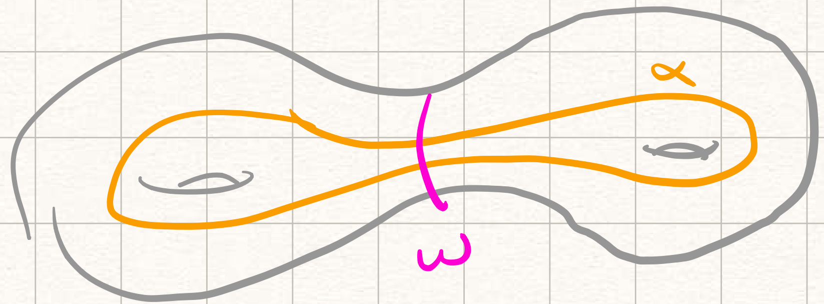
? Don't quite get the invars.

lift of W, invar under γ



is top. inv.

$\alpha \subset S$
crosses ω



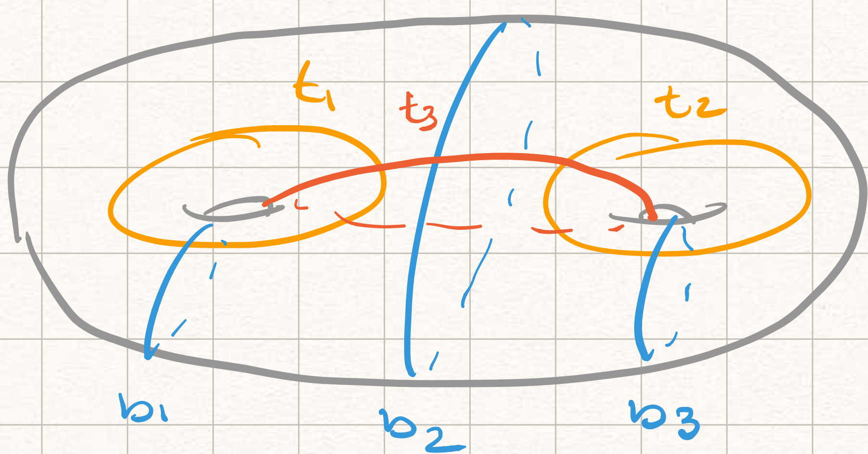
Saraham gives well def'd.
?
:

this gives a well-def'd $T_\omega(\alpha)$.

Markings: A marking μ

$\mu = \text{base}(\mu) \cup \text{transversal}(\mu)$

↑ maximal complex



✱ Stabilizer at most finite b/c whichever curve you twist gets detected by perp.
→ so only get finite elmts.

$\mathcal{M}(S)$ = graph whose vertices are markings, edges are "elementary moves"
↳ ex twisting curve.

$\mathcal{M}(S)$ is : · locally finite (unlike curve complex)

by elementary moves · connected

· $\mathcal{M}(S)/MCG(S)$ finite

Δ = MCG acts w/ finite stab

$$(MCG(S), \text{word}) \xrightarrow{qi} \mathcal{M}(S)$$

I DO NOT GET THIS → $\pi_W : \mathcal{M}(S) \to \mathcal{C}(W)$ makes sense

every marking intersects a surface.

<u>Goal</u>: Understand.

$$\pi : \mathcal{M}(S) \to \prod_{W \subseteq S} \mathcal{C}(W)$$

<u>Properties</u>:

bounded distance w/ image space

1) $W \subset V$, then $\pi_W \circ \pi_V \simeq \pi_W$

2) (Berstock's ineq)

$W \cap V \neq \emptyset$, but $W \not\subset V, V \not\subset W\}$

(write $W \pitchfork V$)

if $W \pitchfork W$:

$\pi_W(\partial V), \pi_V(\partial W)$
$\mu \in \mathcal{M}(S)$

α, β two curves, markings or laminations.

$$d_W(\alpha, \beta) \equiv \mathrm{dist}_{\mathcal{C}}(\pi_W(\alpha), \pi_W(\beta))$$

$$\min(d_W(\partial V, \mu)), d_V(\partial W, \mu)) < C_1$$

$\Uparrow$ univ ct.

we'd like to understand ~~this~~.

3) Bounded geod image thm

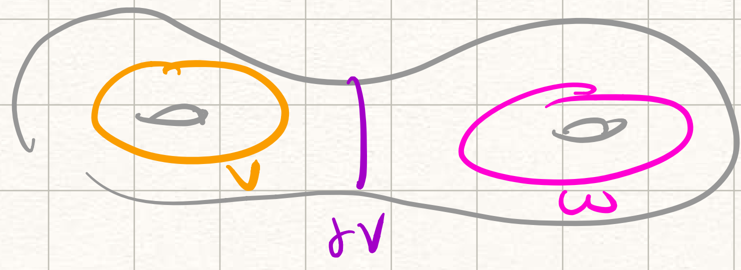
$V \subset W \subseteq S$, $g = (\ldots, g_i, g_{i+1}, \ldots)$ geod in $\mathcal{C}(W)$

If $\pi_V(g_i)$ defined $\forall i$, then

$$\mathrm{diam}(\pi_V(g)) < \underline{C_2}$$

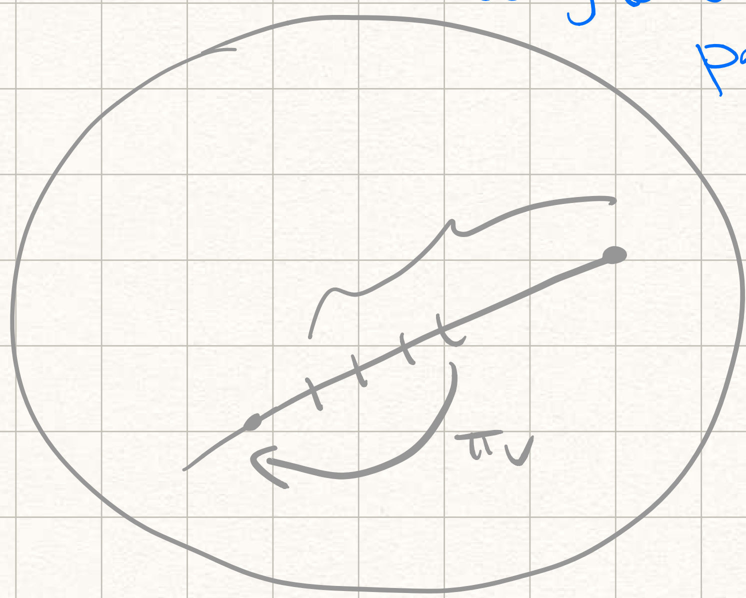
universal bound.

Think of π_V as "visual proj"
link (∂V)



$$\text{link}(\partial V) = \mathcal{C}(V) * \mathcal{C}(W)$$

why is this a free product?



$\mathcal{C}(W)$

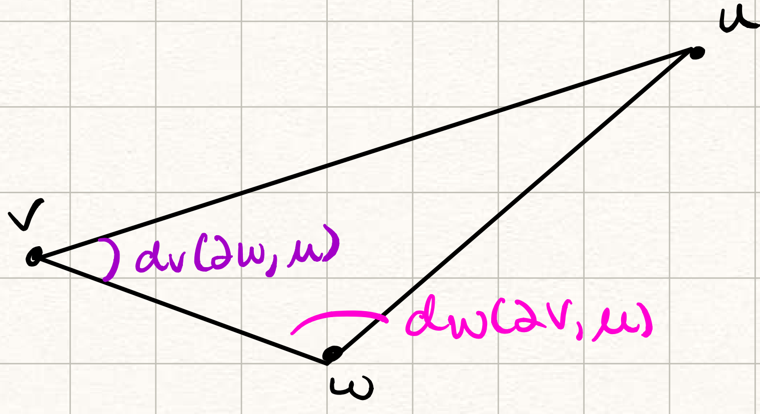
4) Distance formula

$\mu, \nu \in M(S)$

mult & additive error.

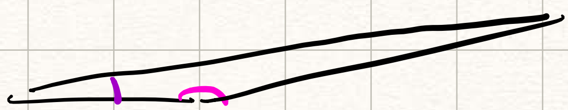
$$d_{M(S)}(\mu, \nu) \simeq \sum_{W \subseteq S} \left\{ d_W(\mu, \nu) \right\}_A$$

$$\{x\}_A = \begin{cases} x, & x \geq A \\ 0, & x < A \end{cases}$$



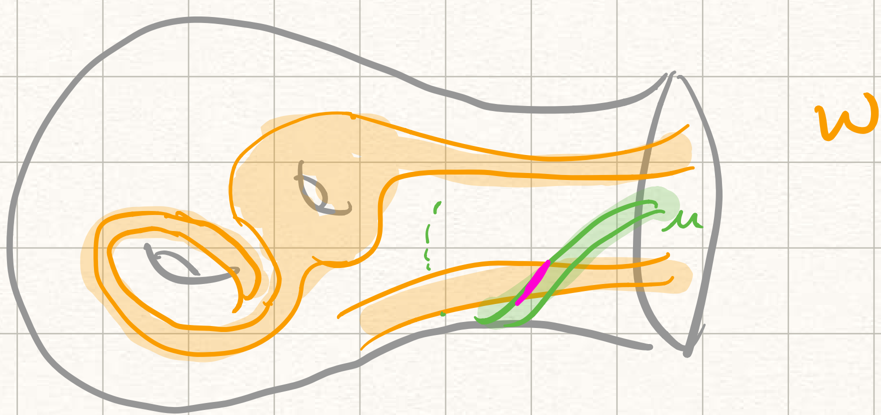
can also have

(review)



but not

Pf:



if $d_V(\partial W, \mu) > 10$, then each arc of $\mu \cap V$ meets ∂W at least 3 times.

$\Rightarrow$ $\exists$ arc of $\mu \cap W$ disjoint ∂V

$\Rightarrow$ $d_W(\partial V, \mu) \leq 1$.

Q: what (coarsely) is the image of π?

Consistency thm (Berk. + Klein. + Mosh.)
(realization) $[\forall c_0, c_1 \quad \exists b$ st:$]$

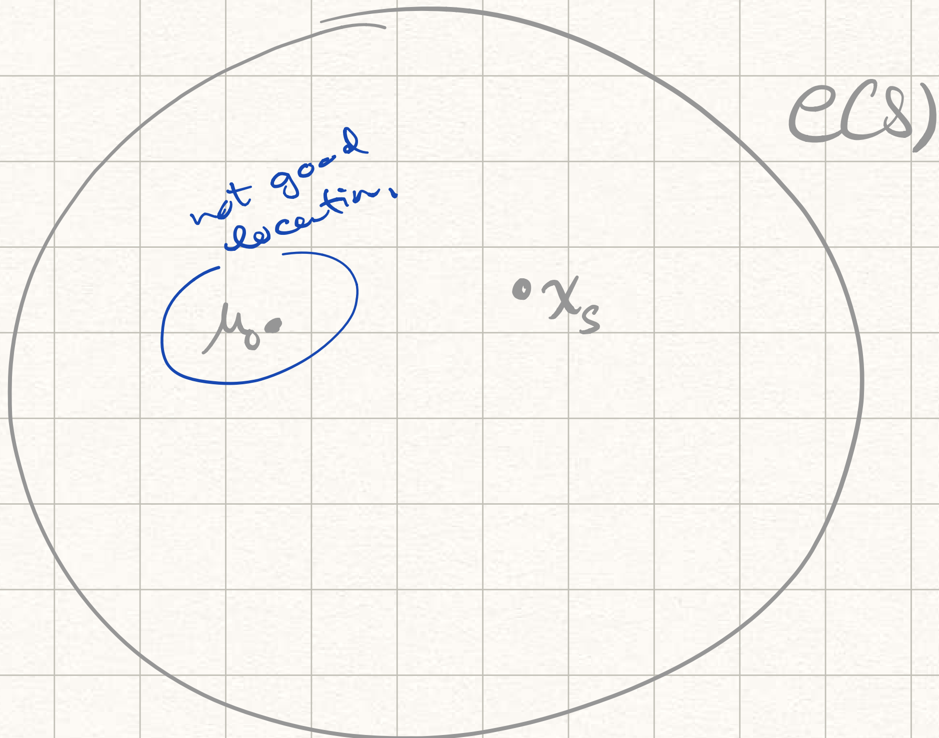
Given $(x_W) \in \Pi\, \mathcal{C}(W)$
$\exists\, \mu \in \mathcal{M}(S)$ st
$\forall W \quad d_W(x_W, \mu) < b$

coarse ver of (1) above

$\Leftrightarrow$ (1) if $V \subset W$, $d_W(\partial V, x_W) > c_0$, then $d_V(x_V, x_W) < c_1$

(2) if $V \pitchfork W$,
$\min(d_V(\partial W, x_V), d_W(\partial V, x_W)) < c_1$

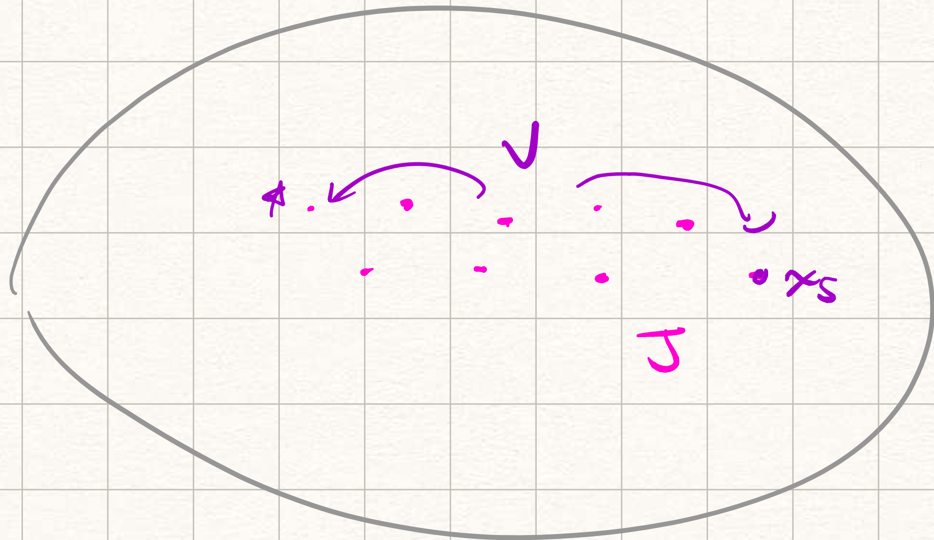
Pf: Given (x_w) satisfying (1) & (2), build μ inductively.



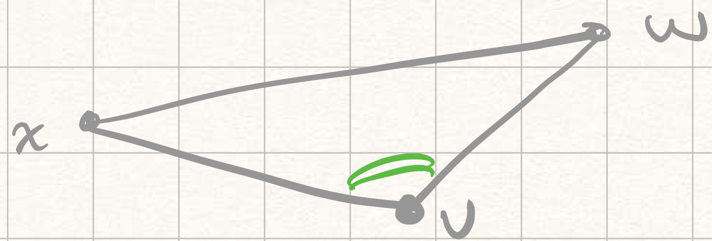
the μ has all the x_w's in it.
but might have obs. V where
μ_0 & x_s seem far.

Look at: $V \subset S$ st $d_V(x_S, x_V) > 100\, c_1$
→ then cond not satisfied.

Berkstock's ineq C(2)
helps to partially order J



Choose $\mathcal{U}$ min in J, let $\partial\mathcal{U}$ be the first part of $\mathcal{U}$.



Say $v < w$ if $d_v(x_v, \partial w) > 100\, c_1$,

$c_2 \Rightarrow$ if $v < w$ then $w \not< v$.

pt: encode ord. struct & then use partial order to argue.