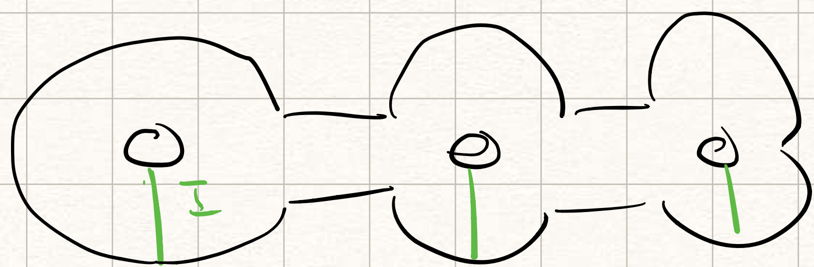
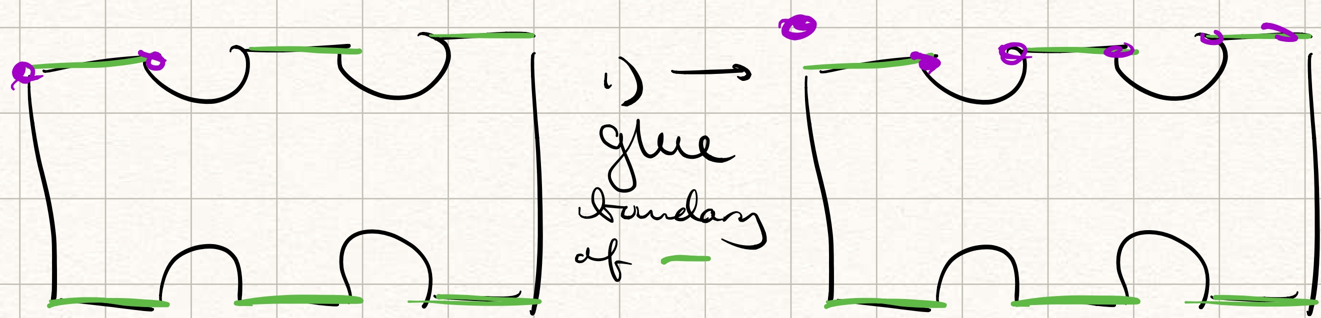
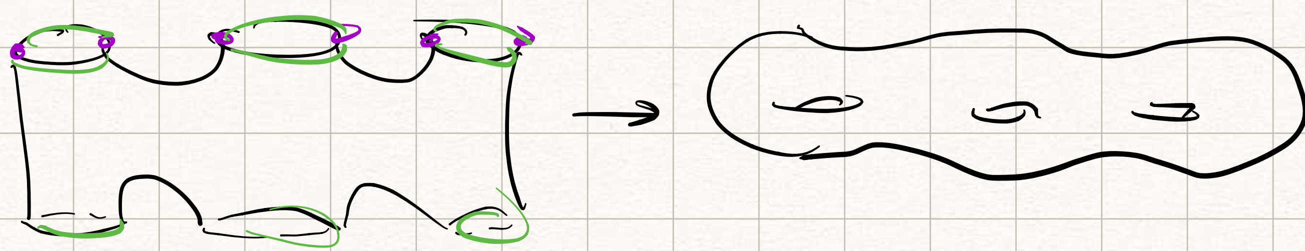
Did review of pic before.

For me: lower dim analogue $S^2 \times I$



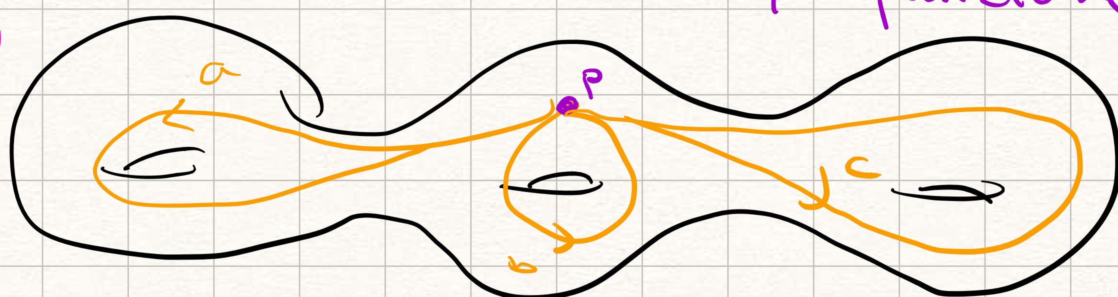




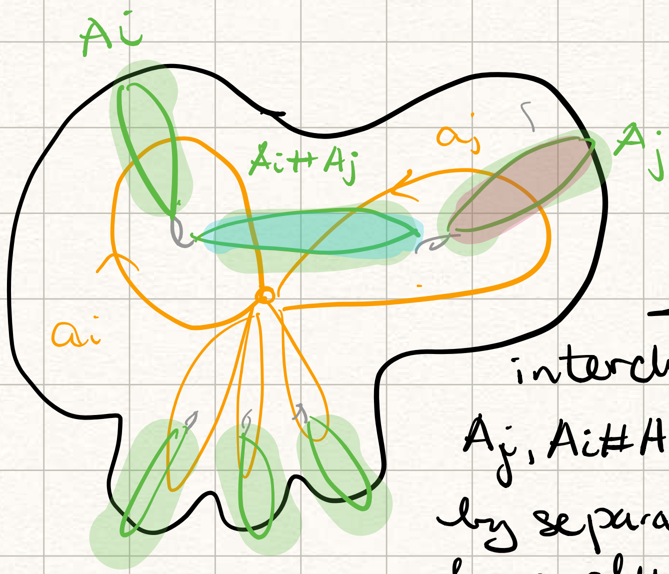
Anyway, for the grps elmt:

p = puncture

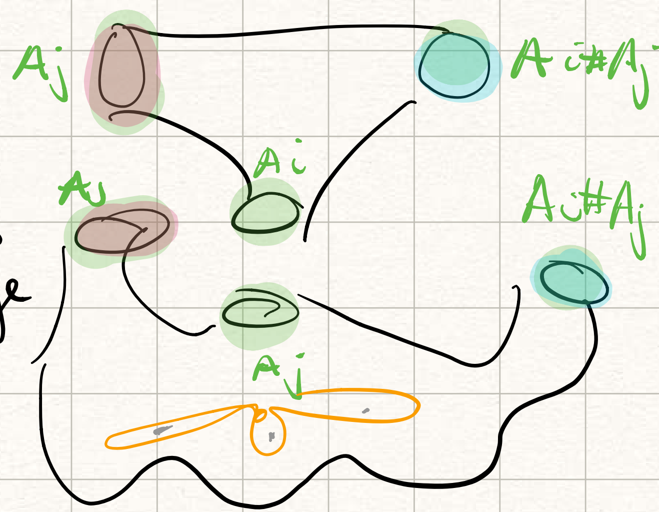
(*)



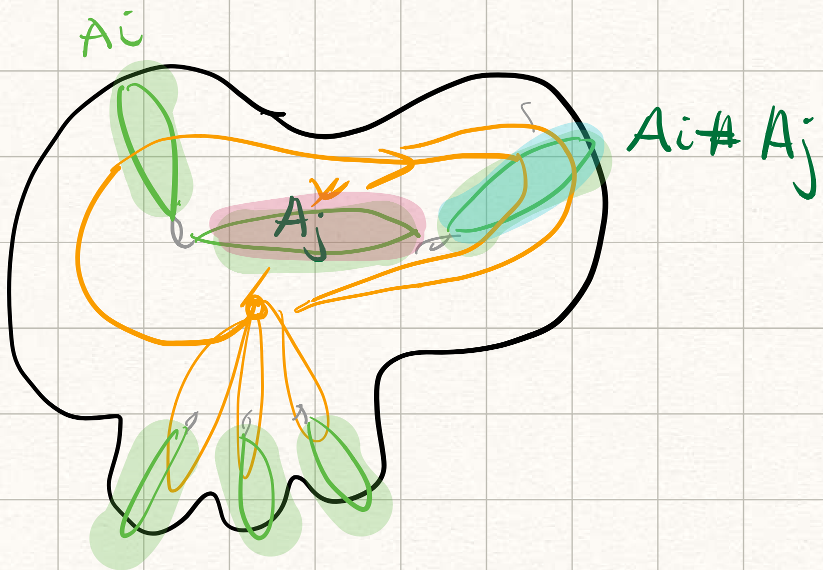
Modeling σ : choose homeomorphism that permute the Spheres A_j & $A_i \# A_j$.



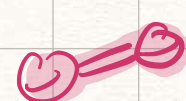
→ interchange $A_j, A_i \# A_j$ by separating & re-gluing



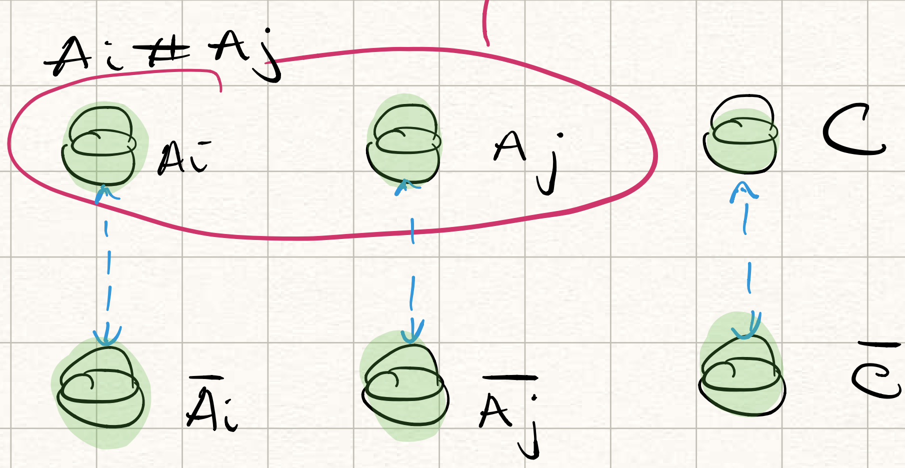
→



$$\left.\begin{array}{l} a_i \mapsto a_i a_j \xrightarrow{i_j^{-1}} a_i a_j^{-1} \\ a_j \mapsto a_j^{-1} \mapsto a_j \end{array}\right\} P_{ij}^{-1}$$

this = 

in A handlebody



$A_j \longleftrightarrow A_i \# A_j$

Since σ, ι_i, P_{ij}, λ_{ij} gen $Aut(F_n)$
So their image generate
$Out(F_n) = Aut(F_n)/Inn(F_n)$

upto homop' equiv.

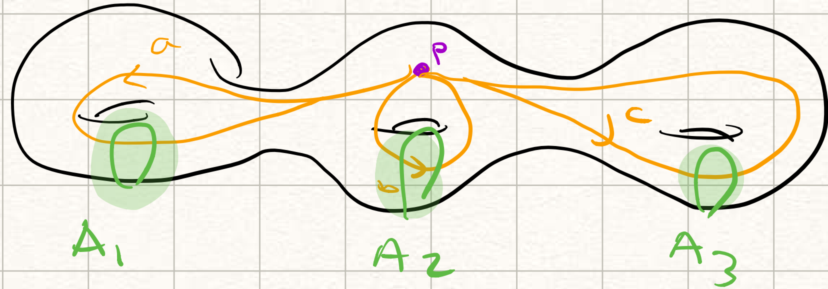
Thm: $\pi_0(\text{Homeo } M_n) \to Out(F_n)$
is onto

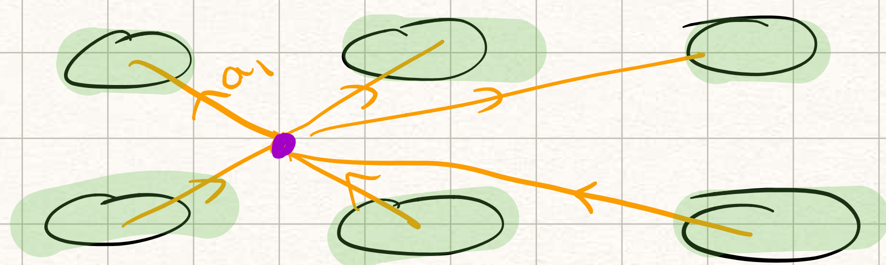
Whitehead (1930s): used this picture
to give an alg to decide
whether $\varphi: F_n \to F_n$ is an
automorphism. $a_i \mapsto w_i$

Recall: Stallings folding gives
another for non-degen cases.

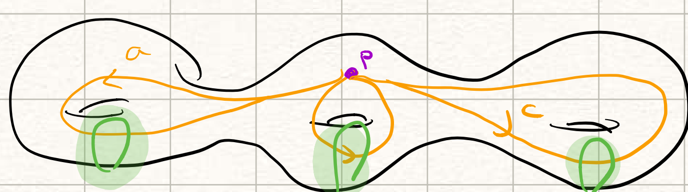
Pf: It's an aut $\iff \{w_1, \dots, w_n\}$ form a basis for F_n.

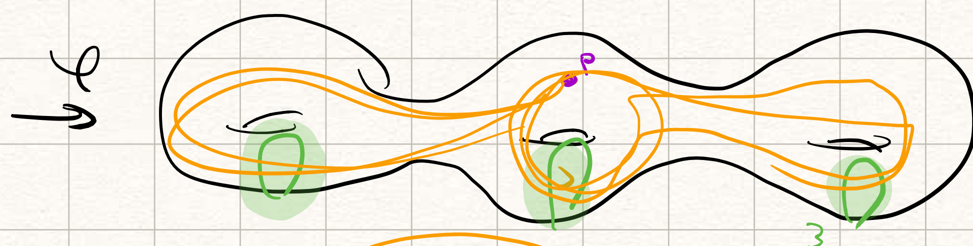
If φ is an aut, model it by homeo $(M_n, p) \to (M_n, p)$ which fixes bpt.

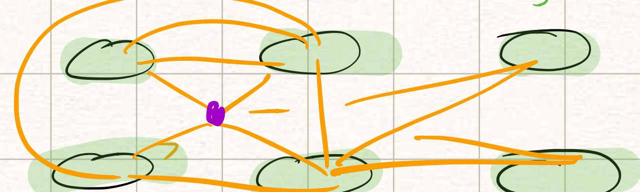




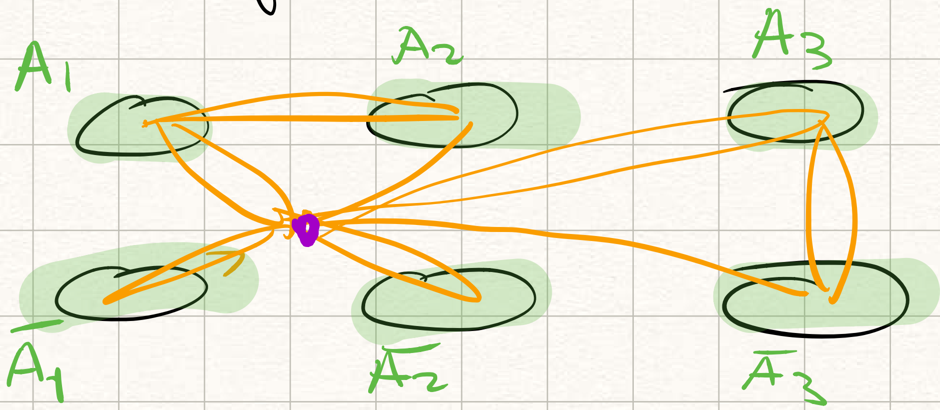
So:







Now "squint"

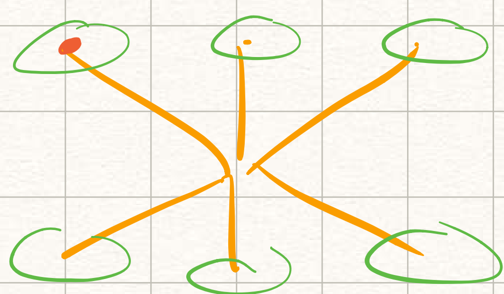


This is called Whitehead's stargraph

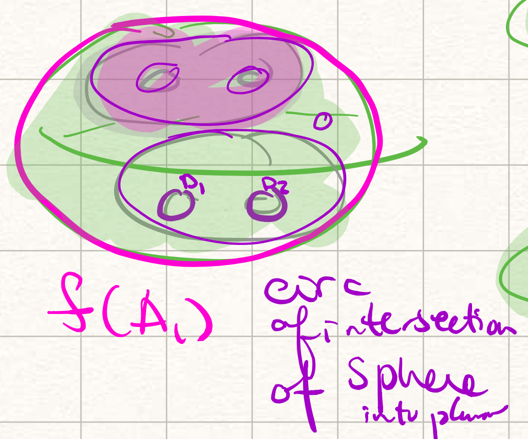
ie, there is an edge A_i to A_j

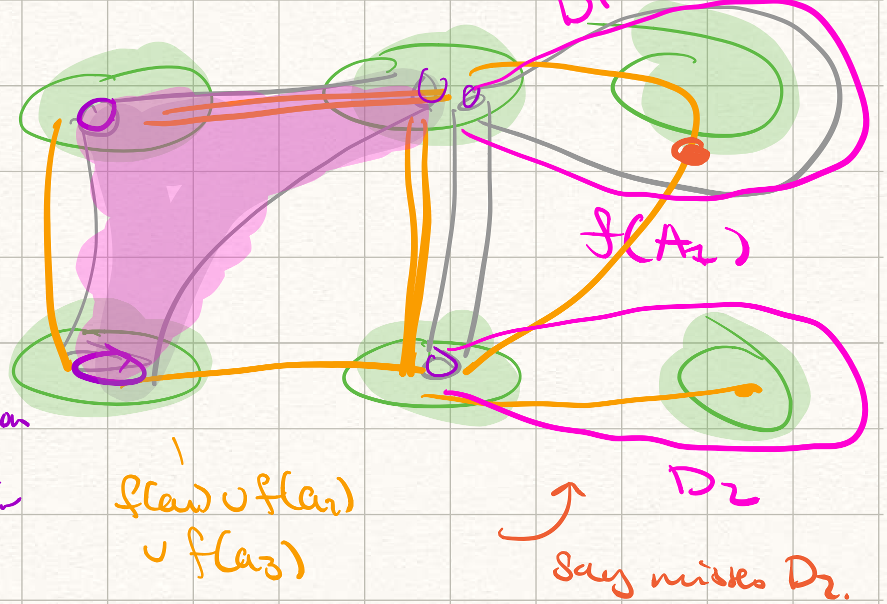
a_j^{-1} a_i $\iff a_i a_j^{-1}$ occurs in some w_k.

Lemma: If $\{w_1, \dots, w_n\}$ is a basis, then the star graph has a cut vertex which is NOT the bpt, or is



Pf: Look at what happens to the spheres A_i under φ.





Isotope $f(A_i)$ to be transferred to the A_i. $f(A_i)$ is a union of planar surfaces, including at least 2 disks ← why?

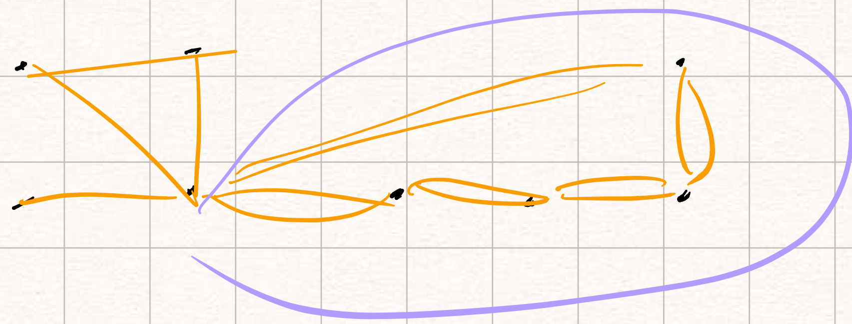
a_1 intersect A_1, not A_2 or A_3, in exactly 1 pt, $\Rightarrow$ must miss at least 1 disk. why?

So $f(a_1)$ intersects $f(A_1)$, not $f(A_2)$ or $f(A_3)$. why?

but $f(a_2)$ doesn't intersect $f(A_1)$
$f(a_3)$ " " $f(A_1)$

$f(a_1) \cup f(a_2) \cup f(a_3)$ misses D_1 or D_2

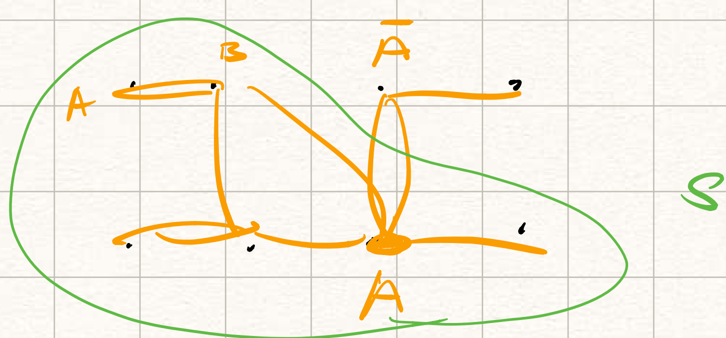
If $\partial D_2 \subset \overline{A_i}$, then A_i is cut vertex. why?



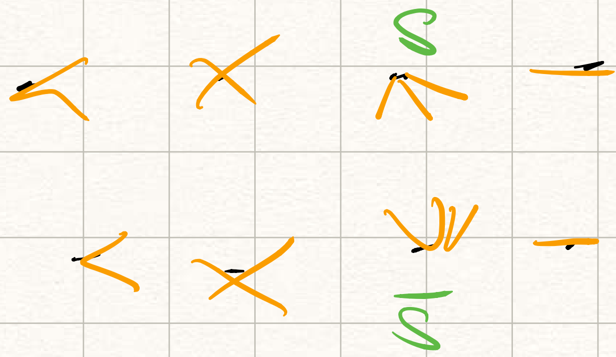
The algo: Given $w_1, \ldots, w_n$ draw the star graph.

i) No cut vertex other than p
$\Rightarrow$ not basis.

ii) Cut vert A (rel cut vertex)
→ Draw specific sphere that separates A from $\bar{A}$.



Take a hom exchanging A w/ S. (✱)
Get star graph, intersecting S in fewer pts than A.



} pt: reducing valence to get basis (they all aut).

If the new star graph doesn't have a cut vertex, then stop.
⇒ can't reduce valence.

Otw, repeat until you get

⇒ have basis.

☆☆: Induces a <u>Whitehead</u> aut of $\pi_1(M_n, b)$.