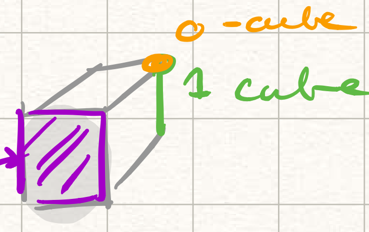
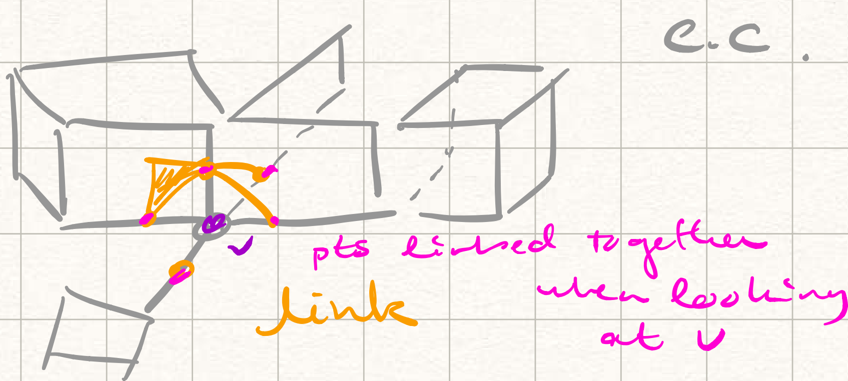
An n-cube is $[0,1]^n$.

Subcube



A cube complex is cell-cmplx built by gluing cubes along subcubes

e.c.



The link of a 0-cube $x \in X^0$ is a simplex complex corr. to "ε-spheres" at x. link(x) has (n-1) simplex for each corner of n-cube at x.

X is non-pos. curved (npc) if link(x) is a flag complex for each $x \in X^0$, ie, a simplicial complex st (n+1) vertices span n-simplex as they are pairwise adjacent.

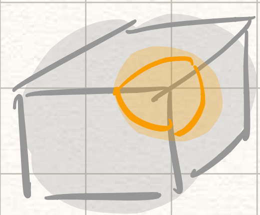
flag-complex $\iff$ simplicial graph

non-ex:

2-skeleton

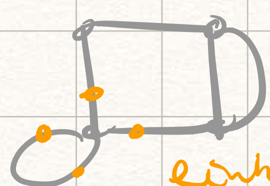
link not filled in b/c no corner.

ex:



3-skeleton.

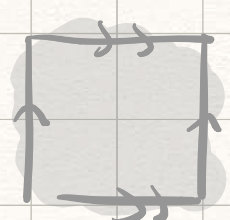
ex: any graph is npccc



link are 0-simplices

exc: any closed surface (except S^2, P^2) is homeo to npccc.

ex:



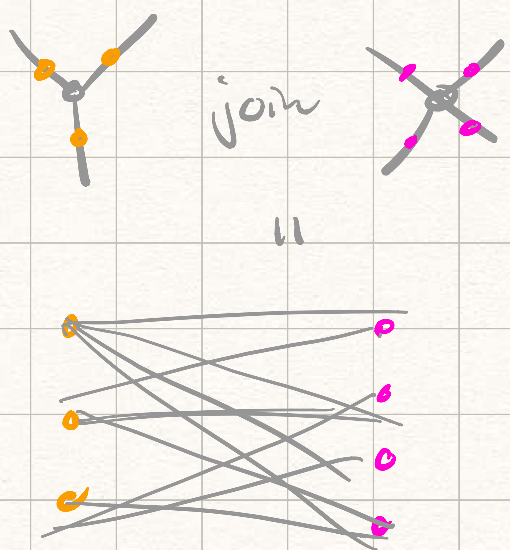
take 2-cubes & glue subcubes (1-cubes).

Lemma: A 2-dim cc or square complex is npc if each each link is a graph of girth ≥ 4

girth(Γ) = infimal len of closed cycle.

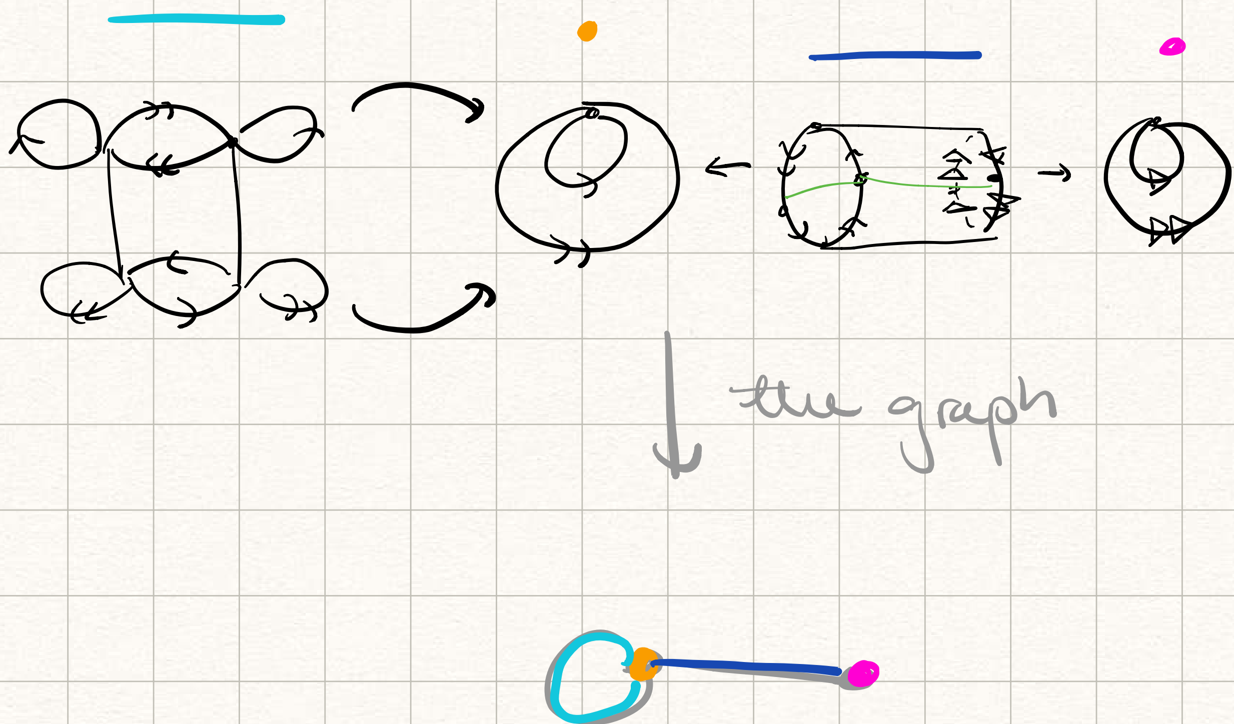
Lemma: The product $A \times B$ of npccc is a npccc.

exc: Since each link of $A \times B$ are joins of links.

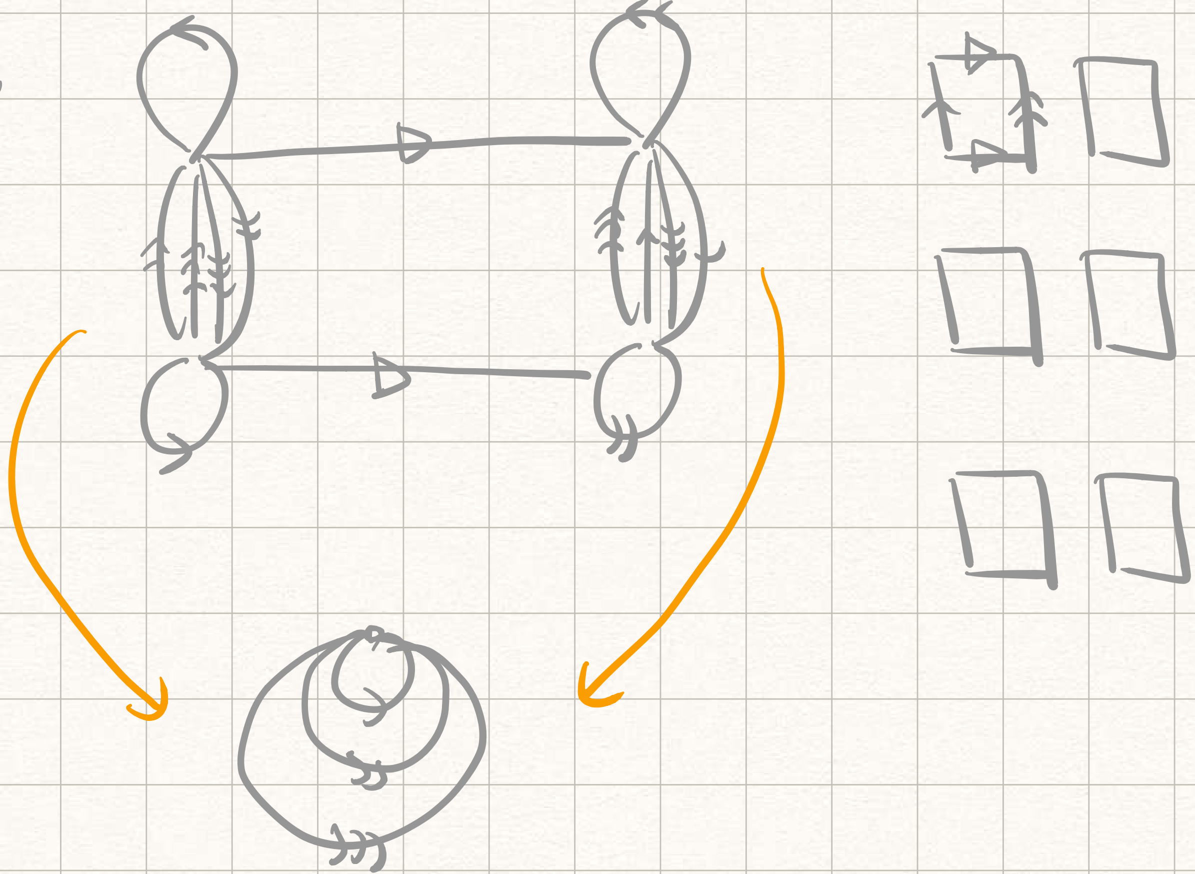


ex: $\mathbb{E}^n = \underbrace{E^1 \times \cdots \times E^1}_{n}$, $E^1 =$ 

ex: A graph of spaces, where
all vertices are spaces
" edges " are space,
and all attaching maps are
combinatorial immersions.



ex:



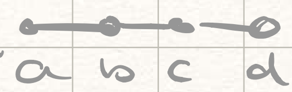
Def: The right-angled Artin grps (RAAGs), $G(\Gamma)$ associated to simplicial graph Γ has pres:

$$\langle v : v \in \text{Vertices}(\Gamma) \mid [u,v] : (u,v) \in \text{Edges}(\Gamma) \rangle$$

ex: $G(\Delta) = \mathbb{Z}^3$

ex: $G($  $) = F_2 \times F_3$

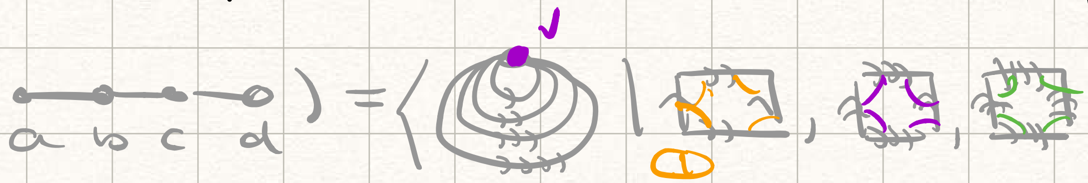
(2,3)-bipartite

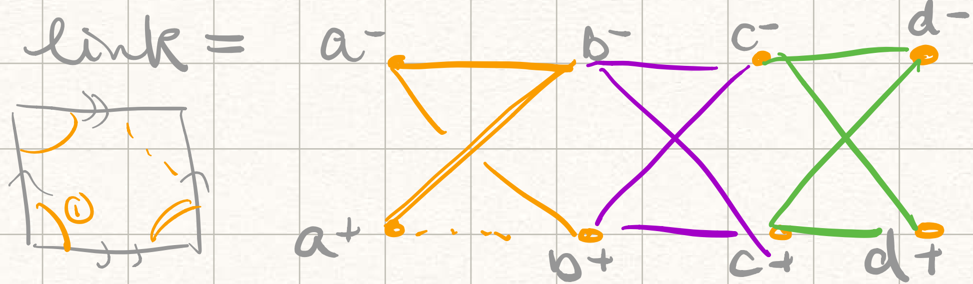
ex: G() ← the girth is ∞

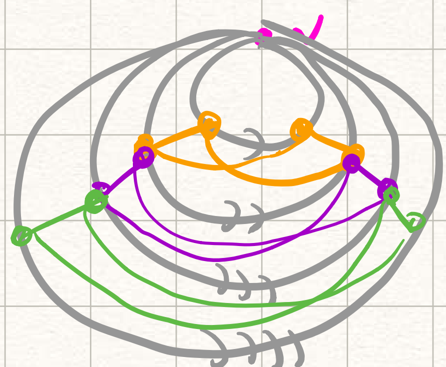
$= \langle a,b,c,d \mid [a,b], [b,c], [c,d] \rangle$

$= \pi_1(S^3 -$  $)$

When girth$(\Gamma) \geq 4$, the std 2-complex of the pres for $G(\Gamma)$ is a npccc.

G( >

link = 



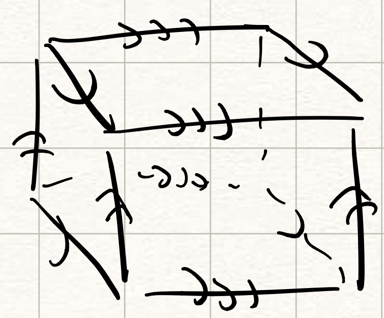
} are these unfilled cubes?

?

In general, $G(\Gamma) = \pi_1 R(\Gamma)$, R is npccc, R is called the Salvetti complex.

Indeed, add an n-cube for each n-click in Γ.

exc:

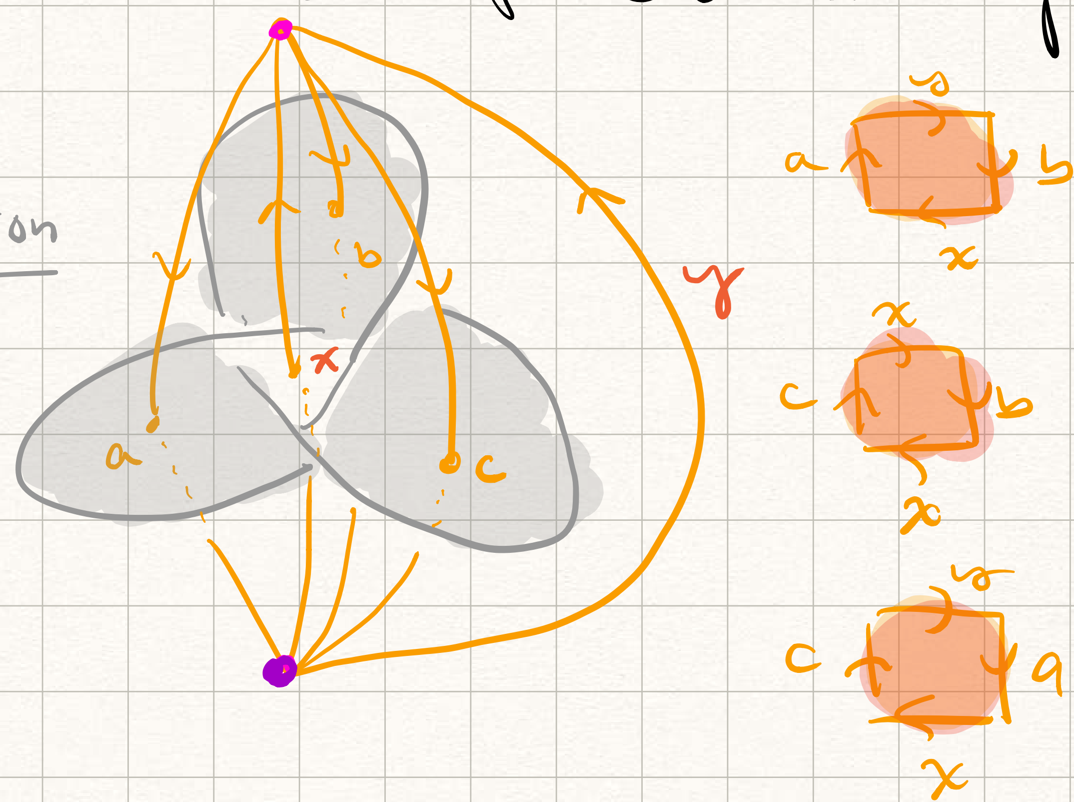


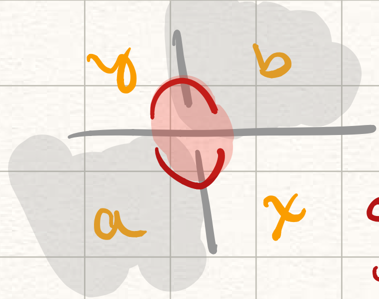
(Another ex)

The Dehn complex X of a knot projection has:

- two zero cube
- 1-cube for each region
- 2 cube for each crossing

the knot projection



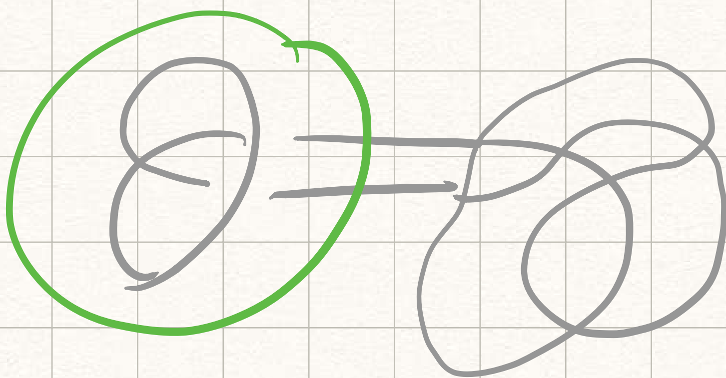


circle, which you pull away.

this is a crossing, there are 3 of thems.

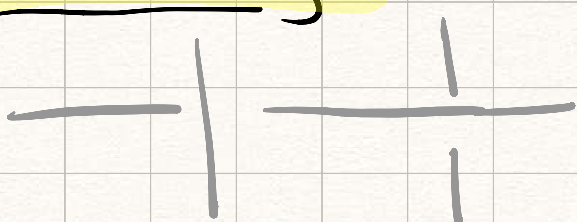
<u>Helpful</u>: the pulling up & down gives orientation of square.

Dehn complex is npcc whenever link is <u>prime</u> or <u>alternating</u>

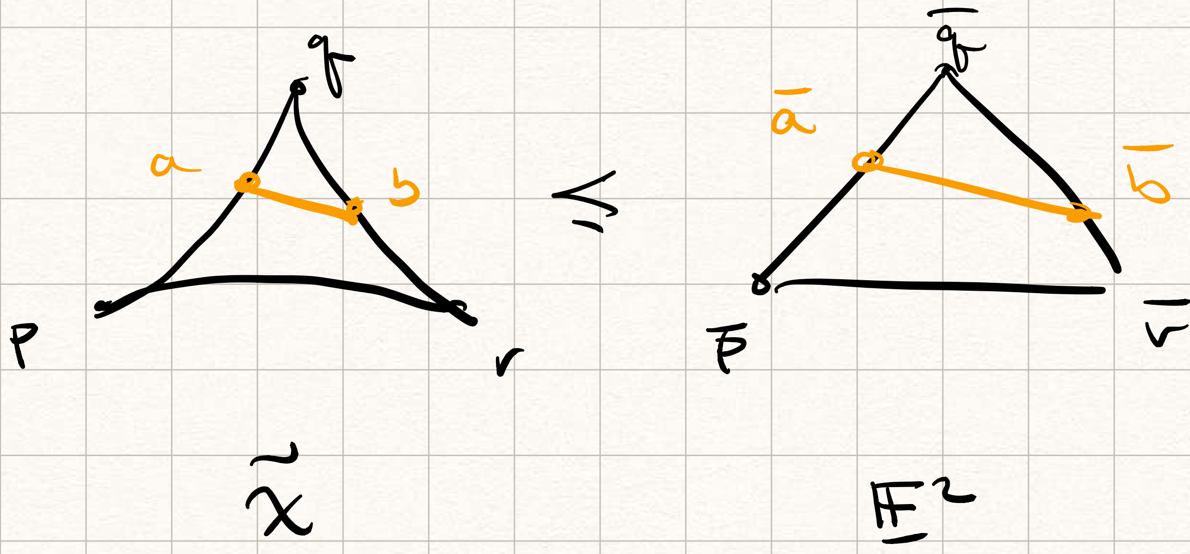


<u>prime</u>:
no circle cuts projection in two interesting pice.

<u>alternating</u>



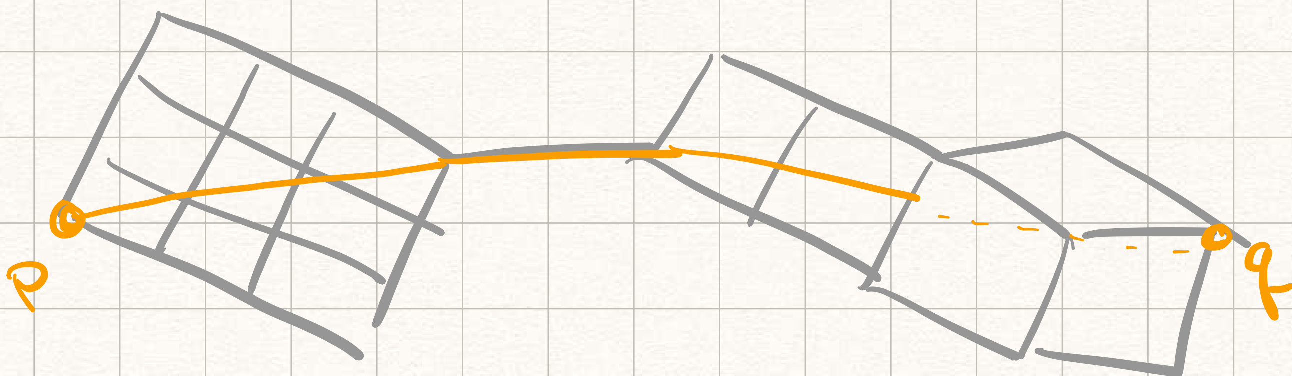
Def: A geod metric space X is a CATCO) space if its geod Δ are as thin as comparison Δ in $\mathbb{E}^2$



$$d_{\tilde{X}}(a,b) \leq d_{\mathbb{E}^2}(a,b)$$

A simply connected npcc is called a CAT(0) cc $\tilde{X}$.

Thm: $\tilde{X}$ has a CAT(0) metric where each n-cube is isometric to $[0,1]^n \subset \mathbb{E}^n$.



geodesics b/w p & q in $\tilde{X}$ is a piecewise cubical paths of min. length (& they exist).