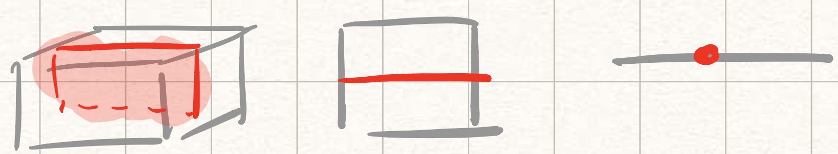
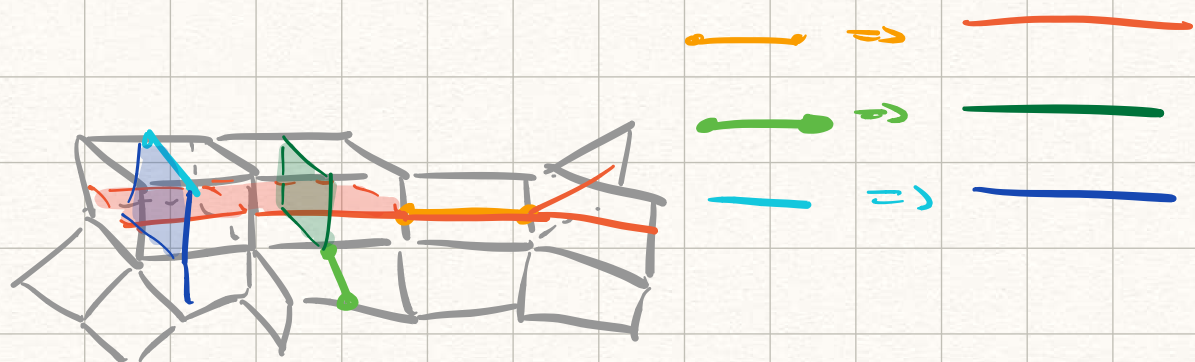
Hyperplanes

A midcube in a cube is a subspace restricting one coord to 1/2.



A hyperplane in a CAT(0) cc $\tilde{X}$ is a subspace whose intersect w/ each cube is either empty or an entire midcube of that cube.



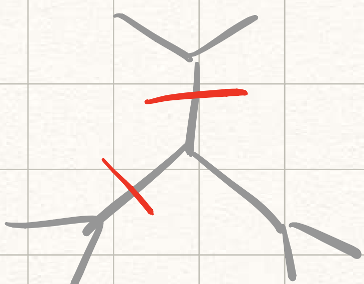
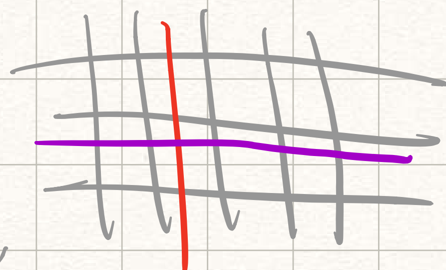
Thm: (Sageev) Every midcube lies in a hyperplane V. (In a CAT(0) cc.)

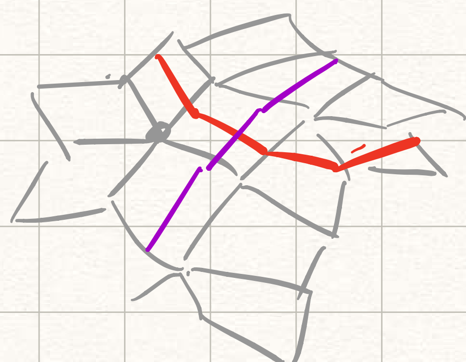
- V separates $\tilde{X}$, ie, $\tilde{X} - V$ has two connected components.

- The carrier $N(V)$ consisting of all cubes intersecting V has $N(V) \cong V \times [0,1]$.

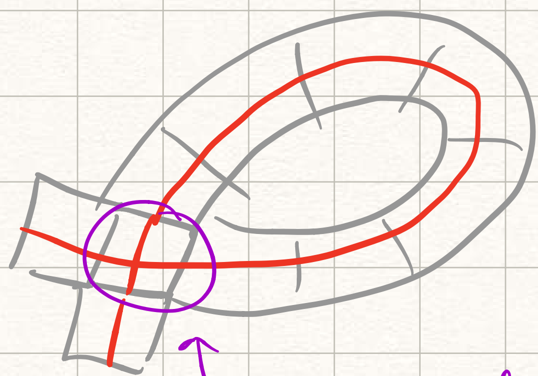
NOT midcubes (and not sitting in cc).

- Hyperplanes are what gives CAT(0) cc their "character".

, ,



When NOT CAT(0):



"immersed hyperplane"

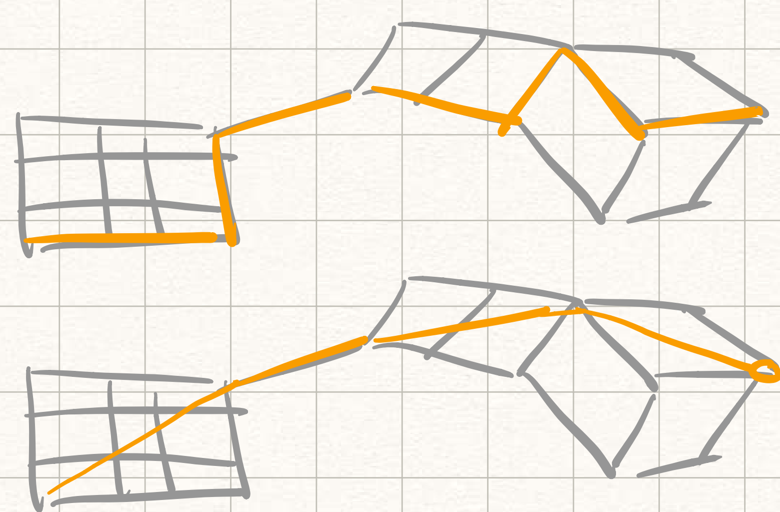
- The metric we will use on CAT(0) cc is the taxicab metric (L¹-metric). It has distance b/w pts equal length of the shortest path that is piecewise parallel to axes w/in cubes.

i.e.:

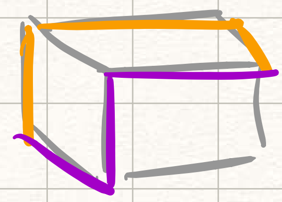
(*)

✓ now

× before



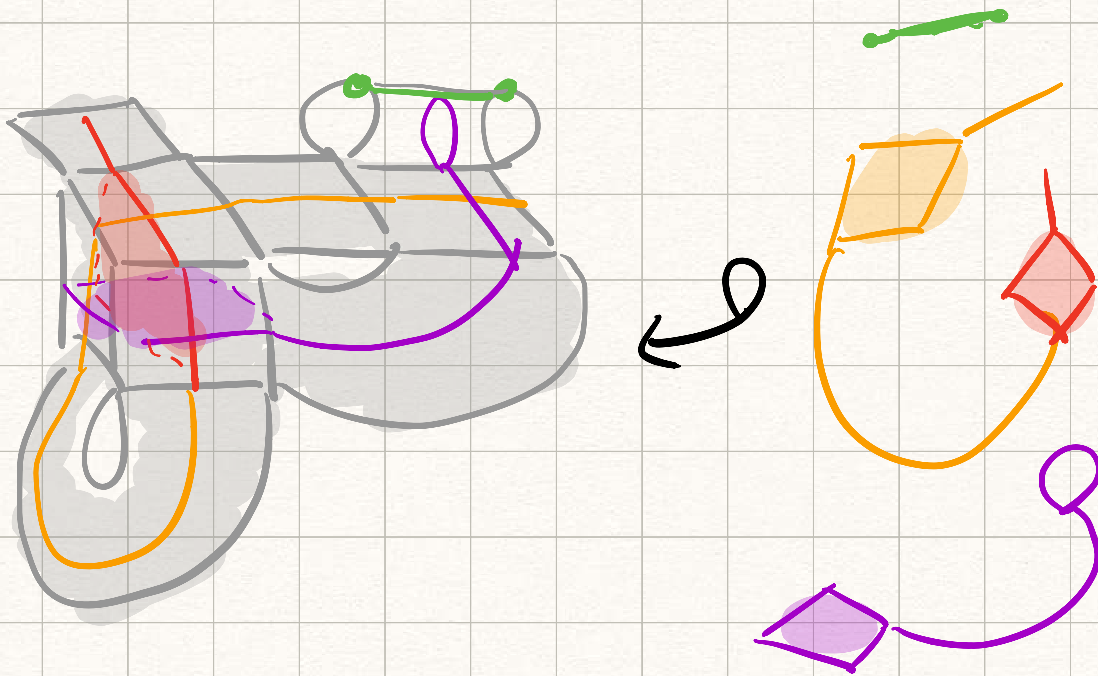
*: Now the geods are non-unique! $\Rightarrow$ <u>NOT</u> CAT(0).



We will focus on metric $\tilde{X}^1$.
L^1-metric induces same metric on $\tilde{X}^0$ as the usual path metric on 1-skeleton.

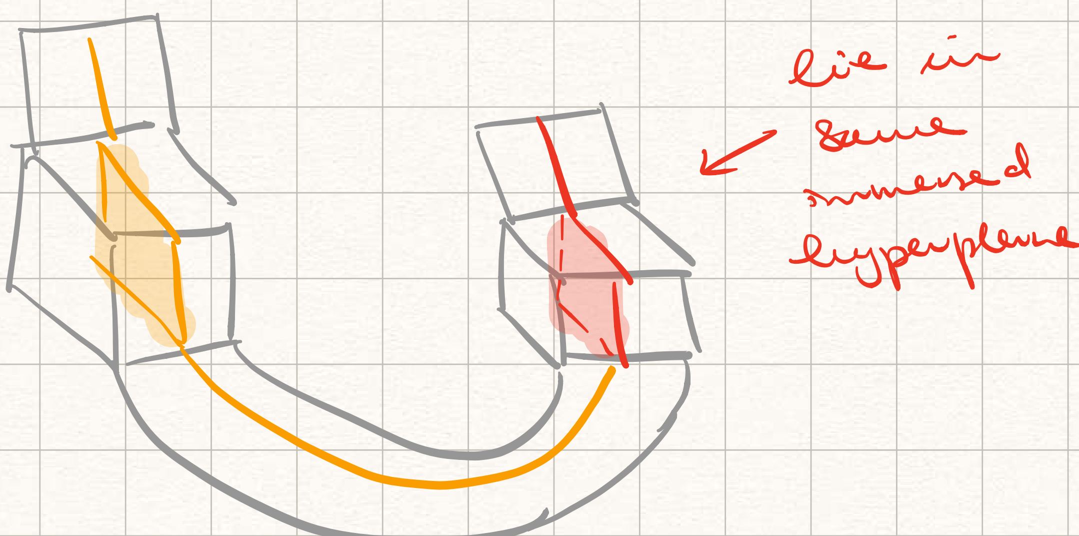
N.B: We're not a CAT(0) metric space, but we just use the CAT(0)-cond on flag complexes.

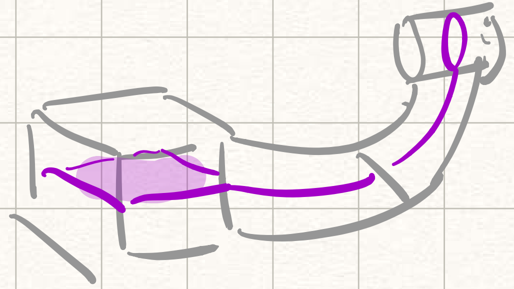
An ==immersed hyperplane== in a npcc is a component of a space obtained by gluing midcubes of X along sub-midcubes.



Basically just separate to get only midcubes.

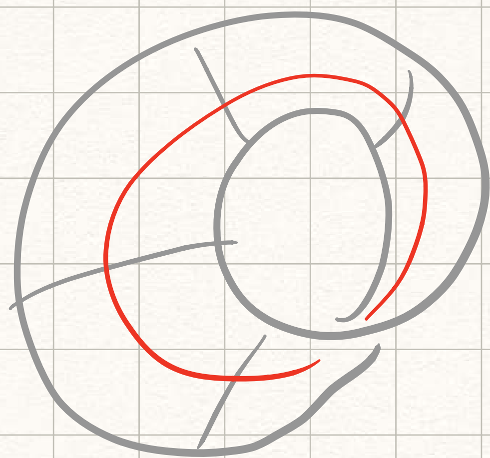
The ==carrier== $N(V)$ of immersed hyperplanes is the cube complex obtained by gluing together ambiant cubes (instead of midcubes).





here's another one.

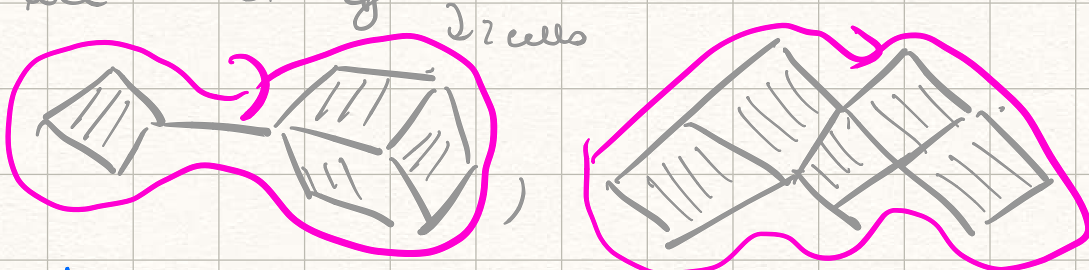
ex:



2 are immersed hyp.

Def: A ==disc diagram== is a compact contractible 2-complex D w/ an embedding $D \subset S^2$.

There are disc. diag



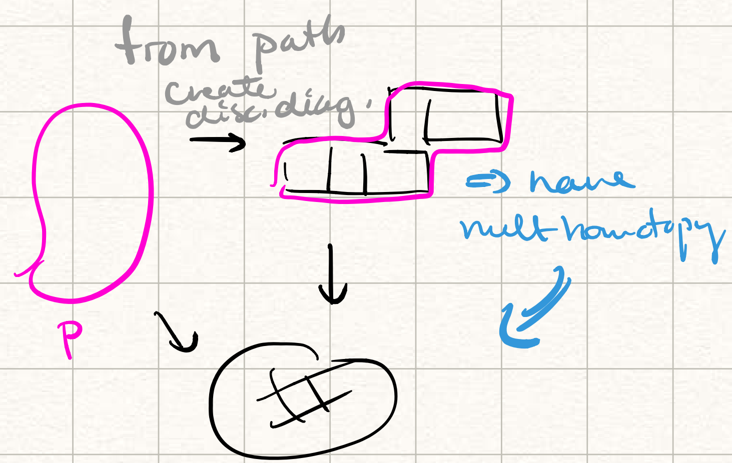
? what does that even mean?,

<u>boundary path of D</u> is attaching map of the 2-cell of S^2 containing ∞. (Think of one more cell outside w/ vertex

at ∞).

Thm (vK) For any closed nullhomotop. comb. path $P \to X$, there exists a combinatorial complex disc. diagram D and a combinatorial* map $D \to X$ st

$$\begin{tikzcd} P = \partial_p D \arrow[r] \arrow[dr] & D \arrow[d] \\ & X \end{tikzcd},$$

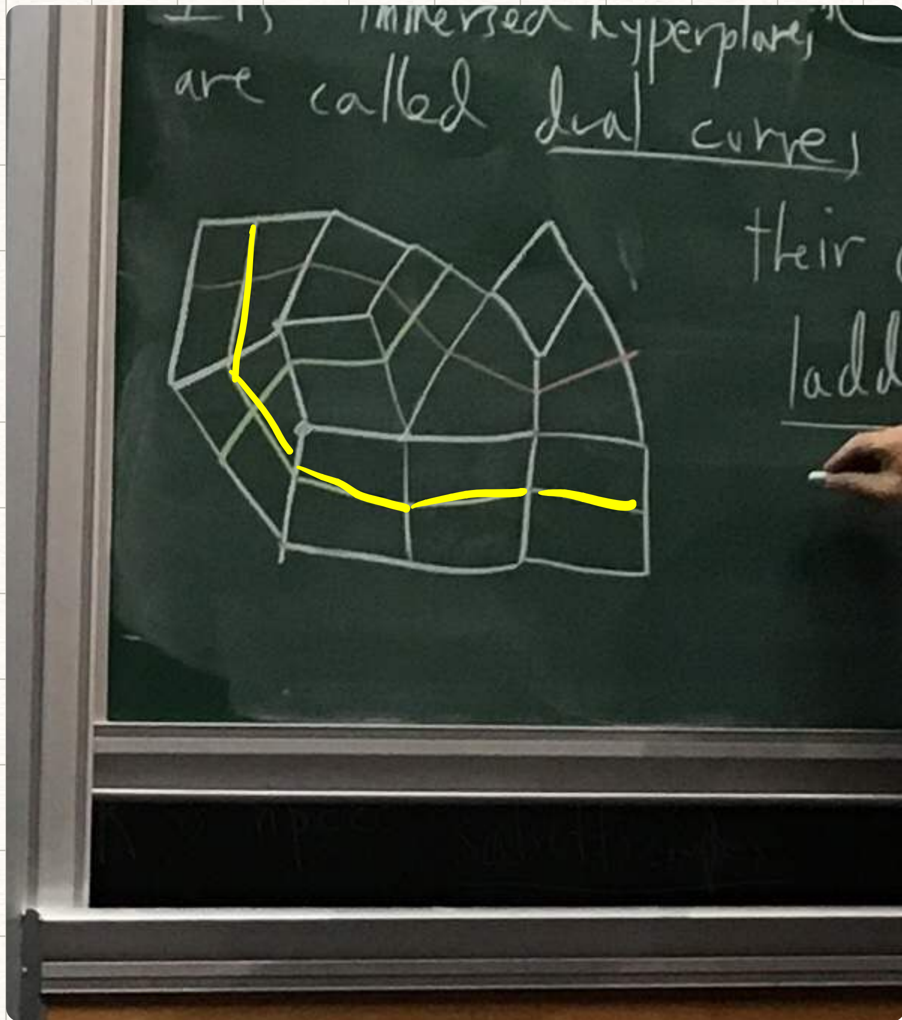


*: n-cells → n-cells homeomorphically

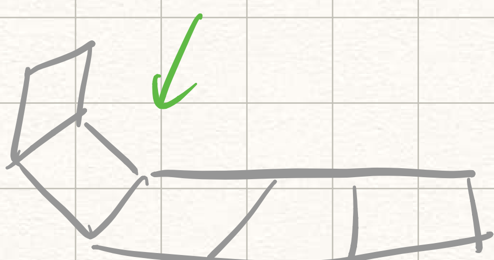
Minimal disk. diagram
have the fewest cells (fewest 2-cells) amongst all diagrams w/ that boundary path.
'ie smallest "area"'

When $D \to X$ is a disk diagram in a cc, D is a **square diagram**.

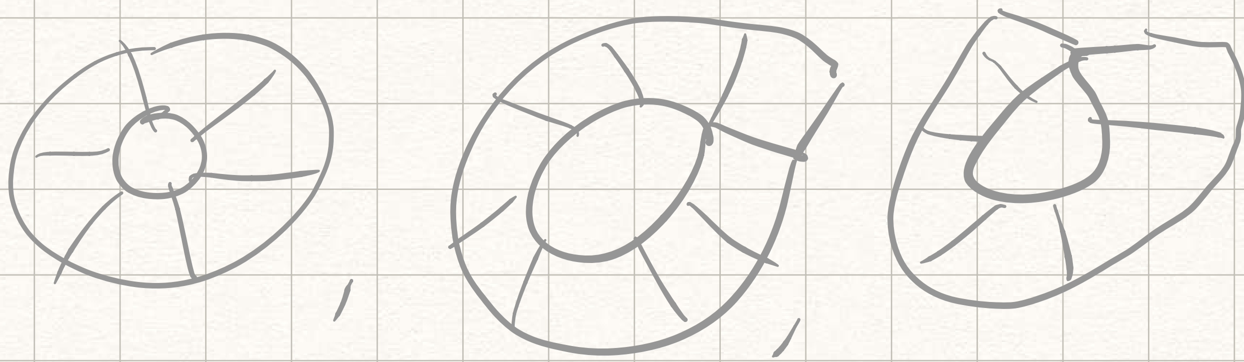
It's "immersed hyperplanes" are called **dual curves**.



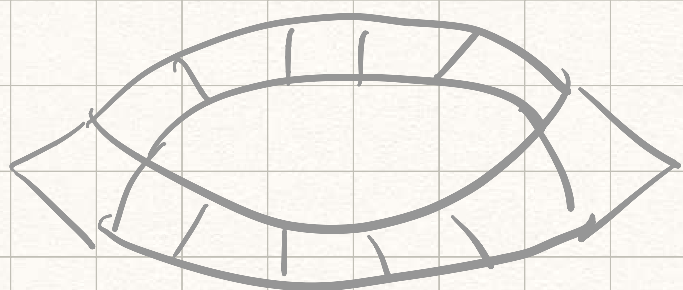
their carriers are called
ladders or cylinders.



Thm: If $D \to X$ is a minimal area disk diagram, then D has no

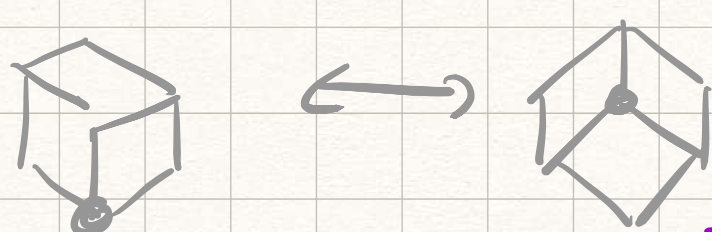


most importantly: no bigons.



Method of pf.

hexagon moves:

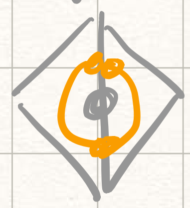
1) 

push the disc diag across the 3-cube.

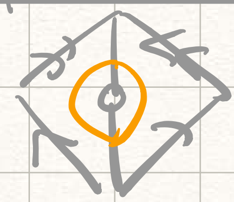
if this on disk diag, means that there's a 3-cube

2) two sq meeting along path of len 2

lk inflag



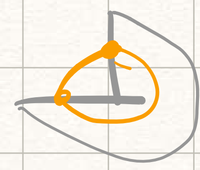
$\Rightarrow$



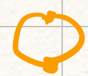
Same edges!

$\Rightarrow$ have to map to the same corner of a sq.

otw?



is that right?

 is NOT simplicial

$\longrightarrow$ replace by

b/c these two have have the same boundary path.