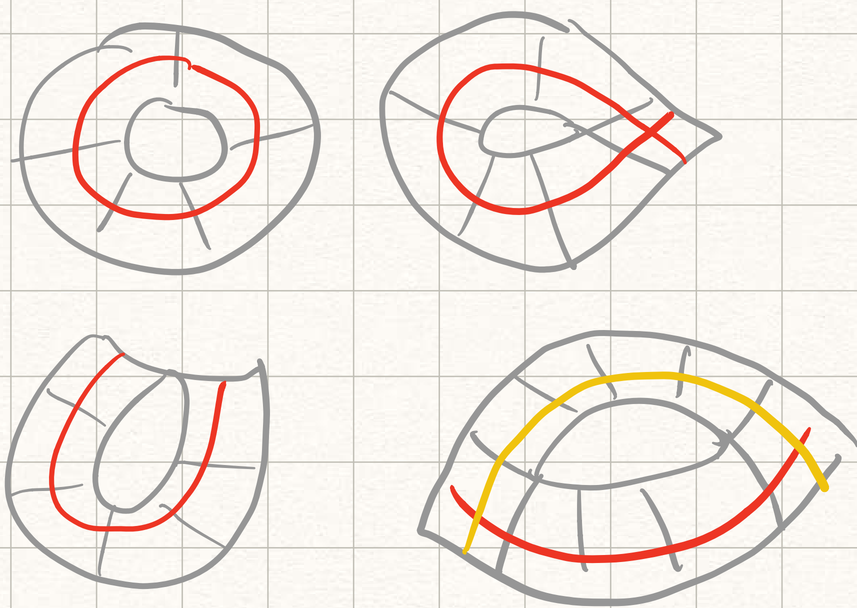
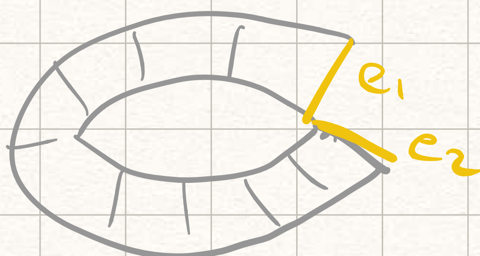
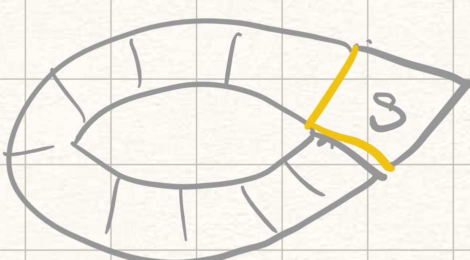
Thm: If $D \to X$ is min. diagram in a npccc, then D has no



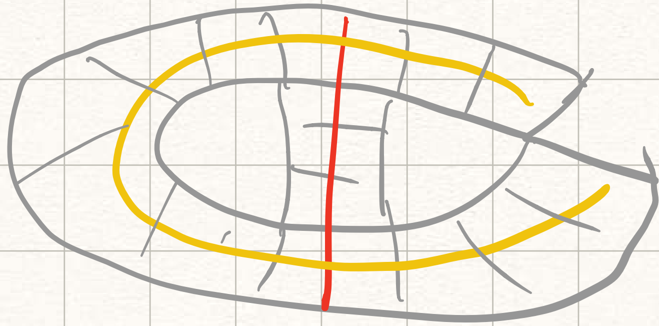
Furthermore, if



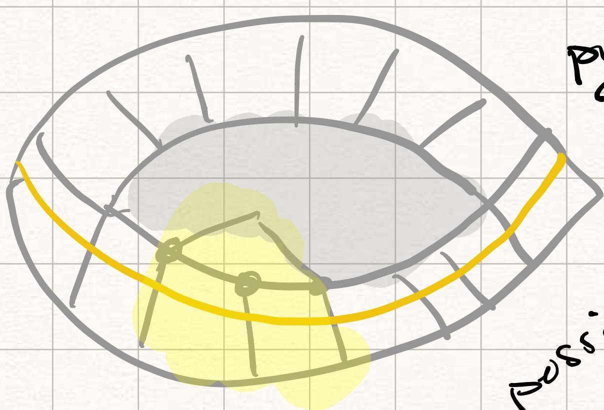
lies in the diagram, then $\exists$ square s at e_1, e_2



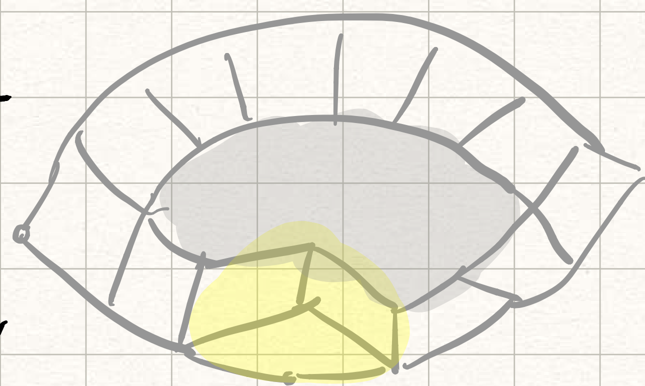
Method of pf: hexagon moves.



If any of pathology exists, then have bigon.

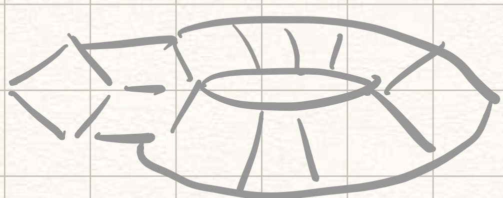


push alt cube $\longrightarrow$

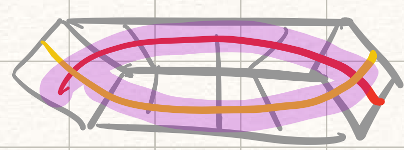


possible..

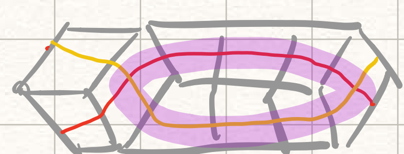
removed squares from interior.



...



... $\Rightarrow$

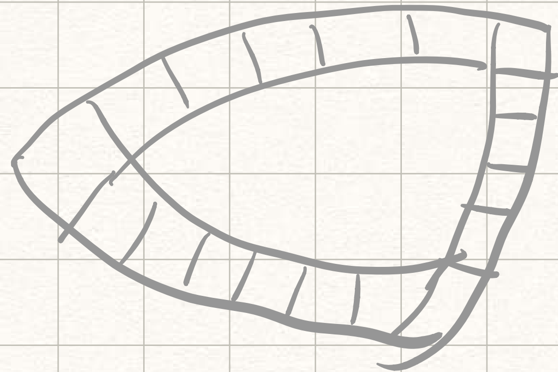


reduce area by 2. from the bigon.

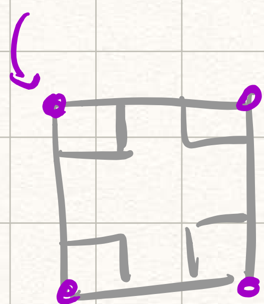
then keep going.

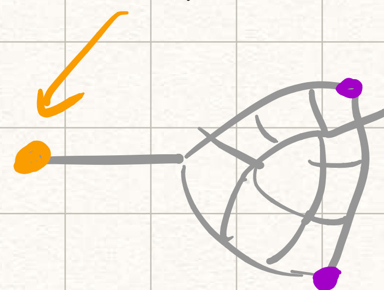
Prop: Let $D \to X$ be a min. len. diagram w/ npccc. Either:

- D is "trivial",
- or D is "triangular":



- or $\#$ square corners $+ 2\,\#$ spurs ≥ 4.

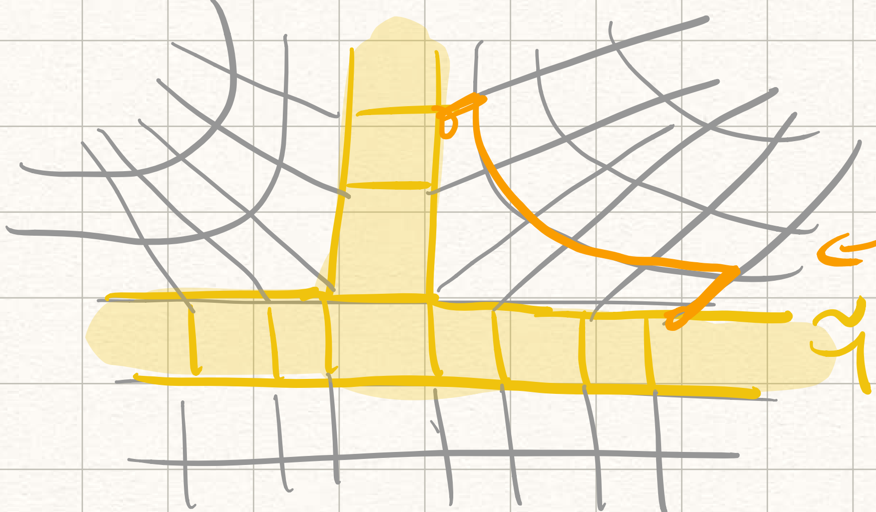




Convexity

$\tilde{Y} \subset \tilde{X}$ is convex if $\gamma \subset \tilde{Y}$ where γ is a geodesic in $\tilde{X}$ w/ endpts in $\tilde{Y}$.

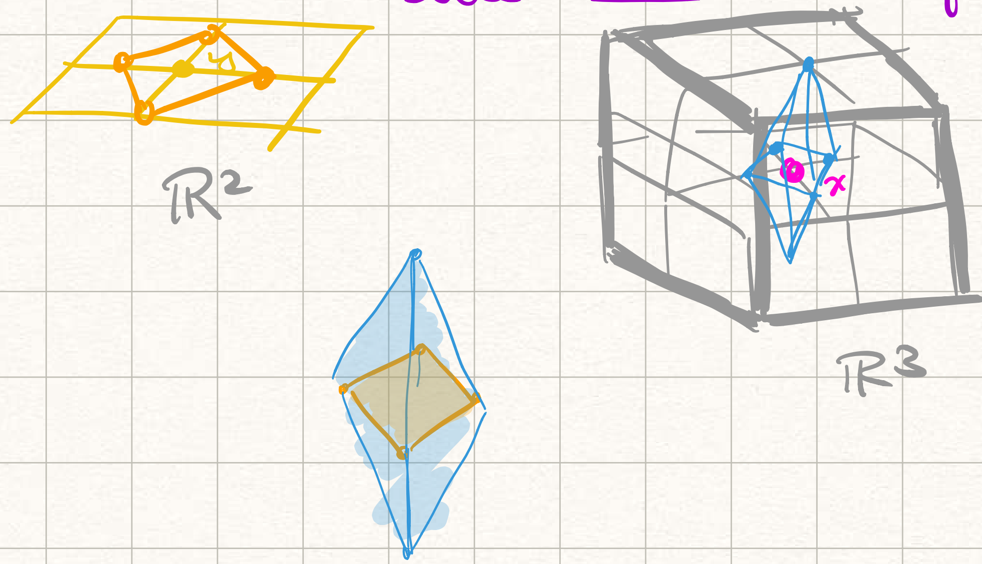
Rem: When $\tilde{Y}$ is a subcomplex, $\tilde{Y}$ is convex when geod γ joining vertices of $\tilde{Y}$ must lie in $\tilde{Y}$, and $c \subset \tilde{Y}$ wherever a corner of c lies in $\tilde{Y}$.



this cannot happen, or orange is NOT geod.

A map $Y \to X$ b/w npccc is a **local isom** if it is combinatorial and for each $y \in Y^\circ$ mapping to x, the map $lk(y) \to lk(x)$ is an embedding as a full subcomplex.

Saying you need to embed the whole link of each vertex.



$lk(y) \subset lk(x)$

Exc: Equivalently, $Y \to X$ is a local isom if it is a comb. immersion (locally inj) st there's no missing corner of a square. ah! that's why the lk is included so.

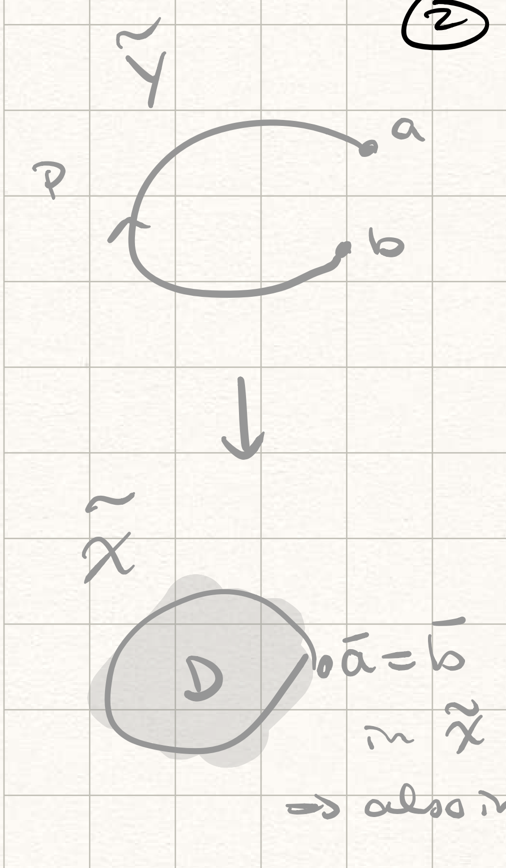
Exc: Any comb. immersion of a graph is a local isom.

Thm: Let $\varphi : Y \to X$ be a local isom w/ X, Y connected npccc.

1) $\tilde{\Phi} : \tilde{Y} \to \tilde{X}$ is injective.
2) hence: $\varphi_* : \pi_1 Y \to \pi_1 X$ is injective.
3) $\tilde{Y} \hookrightarrow \tilde{X}$ is a convex subcomplex.

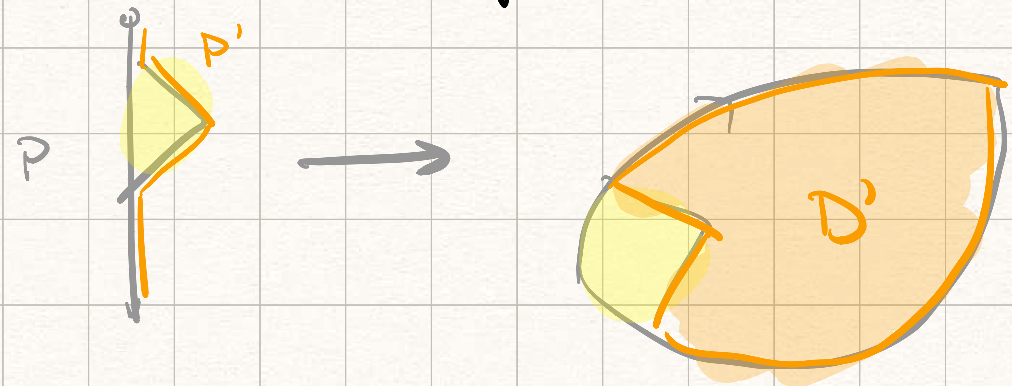
<u>Pf</u>: ① Suppose $a \neq b \in \tilde{Y}^0$ are are mapped to the same pt of $\tilde{X}^0$.

② Choose a path $P \to \tilde{Y}^1$ joining $\bar{a}$ to $\bar{b}$ st $P = \partial_p D$ where $D \to \tilde{X}$ is a disc diagram, and D is minimal among all possible choices.



If D is trivial, then $a = b$.

Else, D has two spurs or sq. corners.

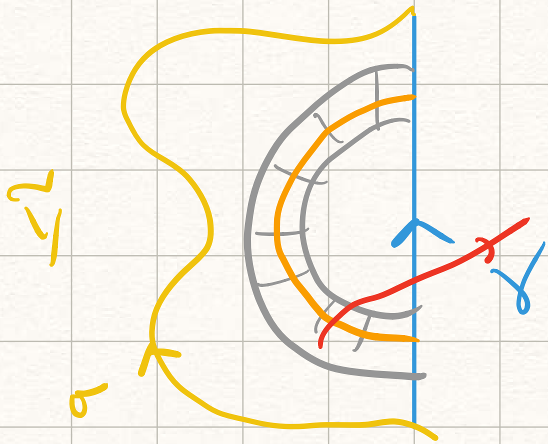


$\Rightarrow$ choose D' smaller area. $\Rightarrow$ Contra!
Do the same thing for spur. $\square$

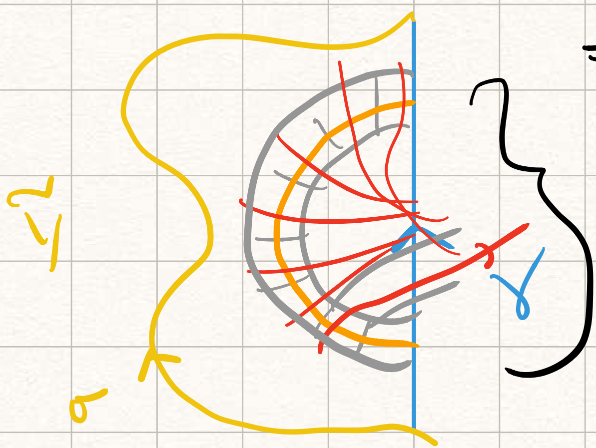
③ Convexity of $\tilde{Y} \subset \tilde{X}$.
Let $\gamma \subset \tilde{X}$ be geodesic from a to b.

Let D be disc. diag. b/w γ and a path $\alpha \to \tilde{Y}$ st D is minimal among all choices (fixing γ)

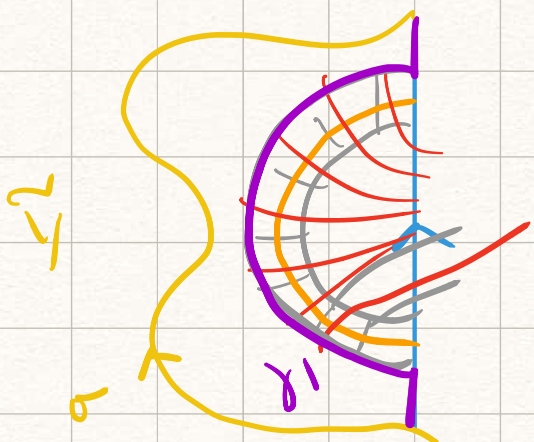
<u>Claim</u>: No dual curve starts & end at γ.



NO BIGONS.



$\Rightarrow$ len γ is at least orange

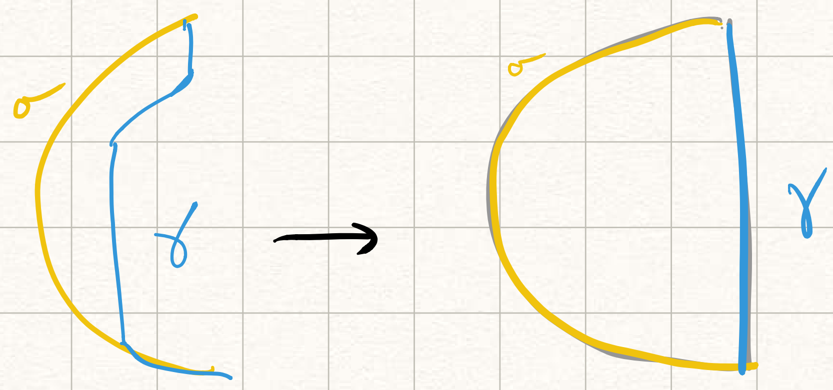


$\Rightarrow$ γ' is shorter. by 2 edges.

$\Rightarrow$ contradiction.

$\square$

If D contains a square:
(Goal: show that $\gamma \subset \sigma \subset \tilde{q}$,
where γ geod w/ endpts in $\tilde{q}$).

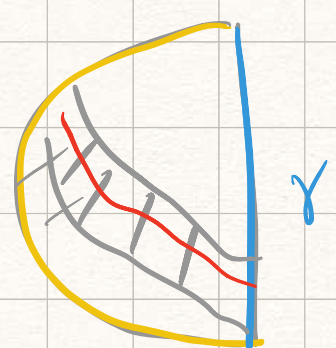


- Choose first edge of γ which isn't in σ.

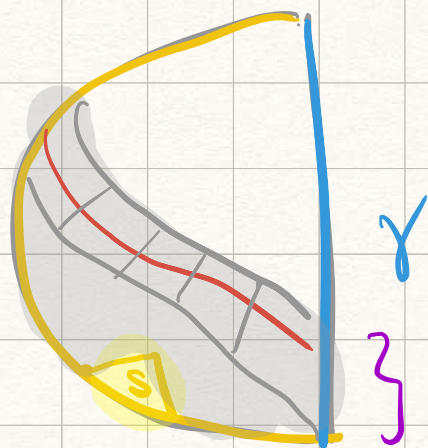
$\Rightarrow$ assume γ leaves $\tilde{q}$ immediately & immediately arrives at $\tilde{q}$.

- Where is the square in D?
Take dual curve & observe that:
  - It can't self cross
  - It can't come back to γ (b/c of claim)

$\Rightarrow$ has to end on σ.



Re-apply proposition to a smaller area:



this entire thing lies in $\tilde{Y}$.

... $\tilde{Y}$ doesn't have any missing sqs $\Rightarrow \gamma = \sigma$.

$\square$