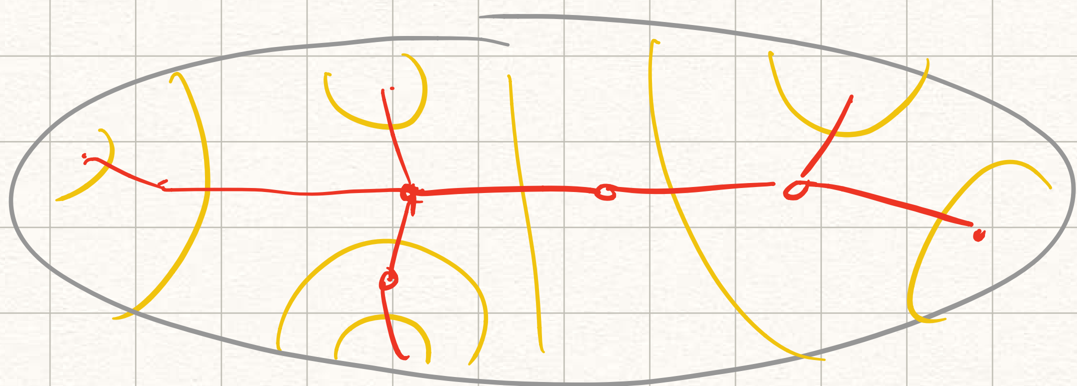
exc: A fg subgrp F_2 is codim 1
$\Leftrightarrow [F_2 : H] = \infty$.

Finiteness Properties of dual

Often, dual arising from the construction is locally infinite, ∞-dim.
- like Bass-Serre tree.

$C \subset A \ast_C B$ has co-dim 1, $C \neq A, B$.

BS-tree = dual to wall sys. from subgrp C.



$C \subset A \ast_{C^t} = D$ is co-dim 1.

ex: Tubular grp, locally finite. but ∞-dimension for Thompson's grp.

Thm (Sageev) Let G be hyp, and $H \subset G$ be codim-1 & q-convex. Then the dual of G is co-compact.

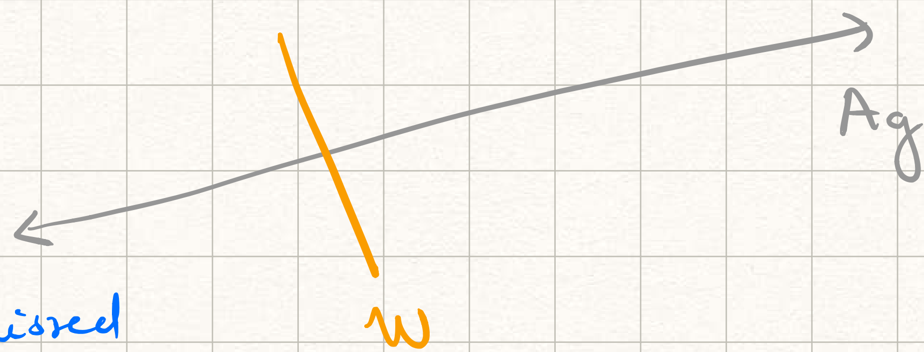
Thm: Suppose that for each non-trivial $g \in G$, there exists a wall that "cuts" g. Then G acts freely on the dual.

Setting: let G act on the wallspace (S, W) by isoms. (properly).

Wall "cuts" g in the sense that for $s \in S$:

ie, $\begin{pmatrix} g^n s \in \vec{W} \\ g^{-n} s \in \overleftarrow{W} \end{pmatrix}$ for $n >> 1$

Often, (S, W) has an extra struct.
e.g. S is a geod. metric space,
g has an axis



missed something.

Thm : let G fg. Suppose for some $A, B > 0$,
$\# C(p,q) \geq A\, d(p,q) - B$
"linear separation"

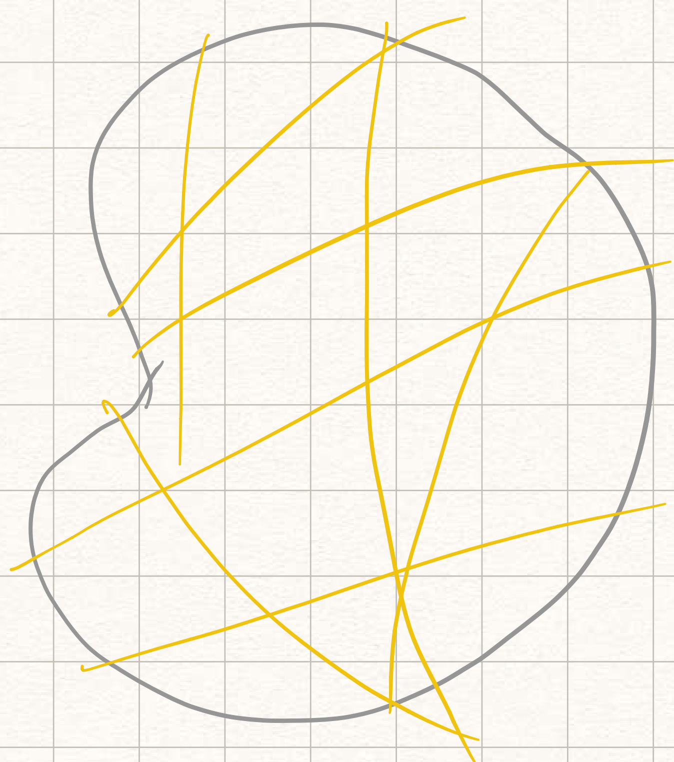
Then the action of G on C has a qi-embedded orbit,
ie, $\exists$ qi embedded

$\gamma \to C$

→ want # wall ∼ dist(p,q).

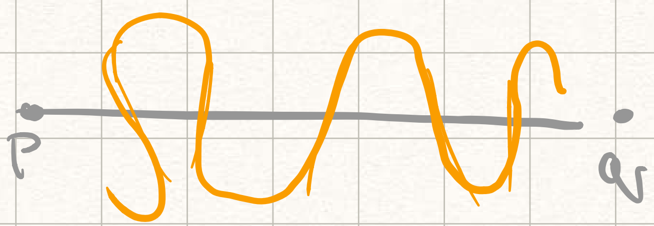
If $\#(p,q) \to \infty$ as $d(p,q) \to \infty$,
then G the map $\gamma \to C$.

Intuition: If the walls cut the wallspace "very well" into little pieces, then the grp will act freely/properly/with finite stabs. on the dual cc.



Unfortunately, we can be easily fooled.

might be reality:

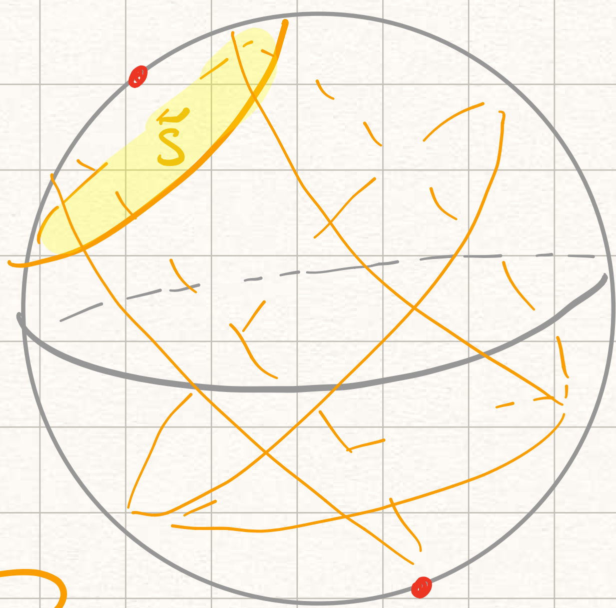


one wall, non-sep.

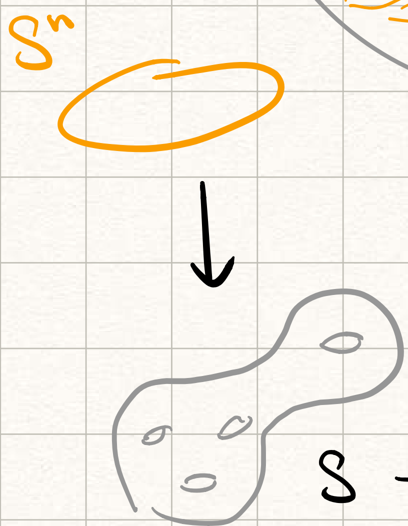
<u>Thm</u>: (Kahn - Markovich)
Let M be closed, hyp, manifold.
$\pi_1 M^3$ contains many q-convex (co-dim 1) closed surface subgrps.

<u>ex</u>: $\pi_1 M^n \subset \pi_1 M^{n+1}$ closed, orientable M's is a 1-codim subgrp.

Using sufficiently many of these surface subgrps, we obtain a <u>free</u> <u>co-compact</u> <u>action</u> of $\pi_1 M$ on a CAT(0) cc.



re-listen.

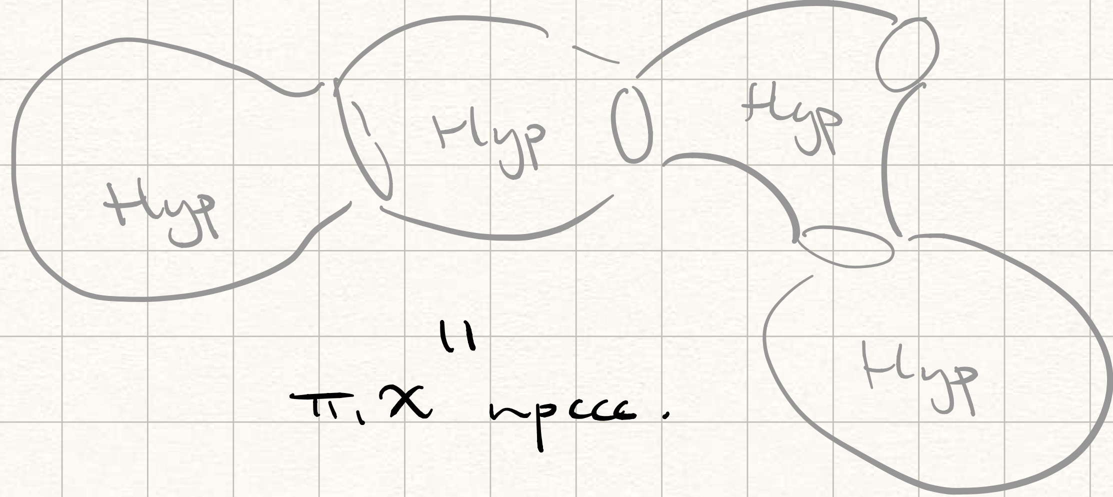


$$\begin{tikzcd} & \tilde{M}^3 \arrow[d] \\ S \arrow[r] & M^3 \end{tikzcd}$$

Conj : Every closed hyp. M^n is π_1(of a compact npccc).

Thm Most compact 3-manif has $\pi_1 M = \pi_1 X$ npccc. (not necessarily compact).

exceptions : Nils, Sols, $\widetilde{PSL_2}$, + Certain closed manifs



$\|$

$\pi_1 X$ npccc.

Every fg <u>Coxeter</u> grp $\xrightarrow{\text{is virt in}}$ $\pi_1 X$, X npcccc; however X not compact but is finite dimensional.

G acts properly on a fin. dim on a CAT(0) cc. $\tilde{X}$.

$\exists G' \subset G$ st G acts freely on $G' \backslash \tilde{X}$ is special.

- Hyperbolic arithmetic lattices of simple type also act in CAT(0) cc and are virtually special.

Combination Thm (Dani Wise + Tim Hsu)

If $G = A *_C B$ where G is hyperbolic & $C \subset G$ is q convex and A & B are virtually π_1 compact.

Then G is $\pi_1 X$, X compact cc, npccc.